Experimental investigations on the fly rod deflection

by construction assessor Tobias Hinzmann

Contents:

Preface

Physical investigations tend to be complicated, sophisticated and scientific. It might be the reason why physics often is not attributed to problems of the daily life when people discuss advantages or disadvantages of different tools or ideas. In fact our life is physics. Every object we use during the day, from the spoon in our cup of coffee to the subway that brings us to work is a physical object. The properties of sports equipment are especially promoted by physical investigations. The bathing suite of a swimmer to the parachute of a sky diver, physics contributes to a far ranging optimization of such objects. These studies are often performed in laboratories that are only accessible for the scientists. The open communication of the physical studies is often lacking – because it seems to be “too complicated”.

Myself and Tobias believe that this must not necessarily be the case. It is moreover a question of communication to describe the physical properties of a system in an understandable way. The attribution of dynamic parameters to experimental data, an understandable description of each performed step, clear graphical representations of the trajectories and finally a mathematical approach to an estimation of the experimental data with mathematic expressions that are known from school instead of a complicated differential and integral calculus can do it.

With this work Tobias and I intend to convince the reader about the advantages of the flexible fly rod in comparison to an absolutely stiff fly rod by translating the dynamical properties of the fly rod into simplified mathematical expressions that are based on the mathematics as taught in the 9^{th} or 10^{th} class of school rather than higher mathematics on the university level. Each step is described semantically and the derivations are explained in a way that gives a real imagination of the dynamics of the flexible fly rod instead of presenting a pure final “value” after a long series of formulas which is hard to attribute to the physical object. In such way we believe that the study presented here might be more precious from a socio-scientific point of view than exact calculations based on theoretical physics that often cannot even be described by the theoretical physicist itself who often refers to the calculation process like a black box when asked what the result really means. This would be a poor excuse for lacking phantasy how to correlate real social problems of the daily life to a simplified mathematical treatment. We hope that the study shown here could be understood as a master example how to treat a socio-scientific problem by exact analysis to end up speculative discussions based on semantically controversies that lack any possibility to be proven. It turns out that there is no doubt that the transfer of effort spent on a flexible industrial fly rod into the acceleration of the tip and the fly line must be much better than for an absolutely stiff fly rod.

I thank Tobias for the opportunity to contribute to a common problem of general interest. Our cooperation was a pleasure to me.

Dr. Franz-Josef Schmitt

A) Background

In the following I investigate physically/mathematically the influence of the fly rod deflection on the fly cast based on experimental data.

The deflection of the fly rod is analyzed by a false fly casting sequence of me taken in summer 2012. It was registered by a commercial digital camera taking 30 frames per second. The single pictures of the sequence were analyzed with a computer program.

As fly rod the SAGE 586 RPL+ (2.65 m length) with suitable fly line WF 5 F long belly was used.

For a better understanding of the influence of the deflection the properties of the used fly rod were compared with an absolutely stiff fly rod[1]. The investigated rods therefore exhibit the following properties:

1.) The flexible fly rod has the stiffness of a fast action. The investigations are based on a fly rod of the type SAGE 586 RPL+ (fly line class 5, ~2.65m length).
2.) The stiffness of the absolutely stiff fly rod is assumed to be infinite. (elastic modulus $\rightarrow \infty$), it shows no flection at all during the fly cast.

The comparison of both rods is only possible if they are cast with the same angular speed (velocity of rotation). This assumption is used consequently for the following investigations.

For the sake of simplicity of the investigations some constraints are assumed for both types of fly rods:

– The rotational dynamics of the rod is analyzed. The parallel translation[2] is neglected due to its small fraction of the overall dynamics.
– Both fly rods are cast with the same angular speed (rotational velocity).
– Both fly rods do not carry mass[3].
– The deceleration and post-pulse oscillation of both fly rods is neglected[4].
– The initial and final positions of both fly rods are assumed to be at 40° and 140° relative to the horizontal line.
– A longer fly line (about 20 meters including leader) is cast using a double haul[5].
– The tips of the fly rods accelerate the mass m of the elongated fly lines.

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[1] The indices „f“ and „s“ in the following denote: „f“ = „flexible fly rod“ and „s“ = „absolutely stiff fly rod“.
[2] The translational dynamics is separately analyzed at the end of this study.
[3] Some correlations that are more complex (e.g. deceleration and post-pulse oscillation) can be explained more simple especially by the assumption of massless fly rods. As a consequence some results are strongly idealized what should have a comparable impact for both fly rods. The conclusions that result from the comparison remain valid. It is not visible that the idealization leads to a stronger advantage or disadvantage for one fly rod. I have estimated the influence of the mass of both fly rods in the annex 2.
[4] I will continuously refer to the consequences of the selected constraints during my investigations and make an estimation for the resulting uncertainties.
[5] A further video was taken shortly after the production of the video sequence these investigations are based on. There the same length of fly line is cast without a double haul. (This explains the strong contribution of parallel translation, see section F2.1). The deflection of the fly rod as well as the velocity of the fly line were slightly smaller.

– Both fly rods have the same length L.

The geometrical and dynamic numerical values are evaluated at three positions 40°, 90° and 140° and – if necessary – interpolated between these positions. The figures are used for visualization and are not fully true to scale. The whole investigation is presented aiming to keep and present the physical relations as simple as possible[6].

The results of the investigations are related to the analyzed fly cast sequence[7]. They are gathered by experiment and therefore carrying uncertainty. The uncertainty is discussed at the end of the study – currently it shall be stated that this fact does not question the general derivations of the study.

B) Geometrical investigations

In this section I investigate the geometry of the pathway described by the fly rod and the absolutely stiff fly rod during the fly cast.

B1) Flexible fly rod

The following geometrical relations hold for the fly rod:

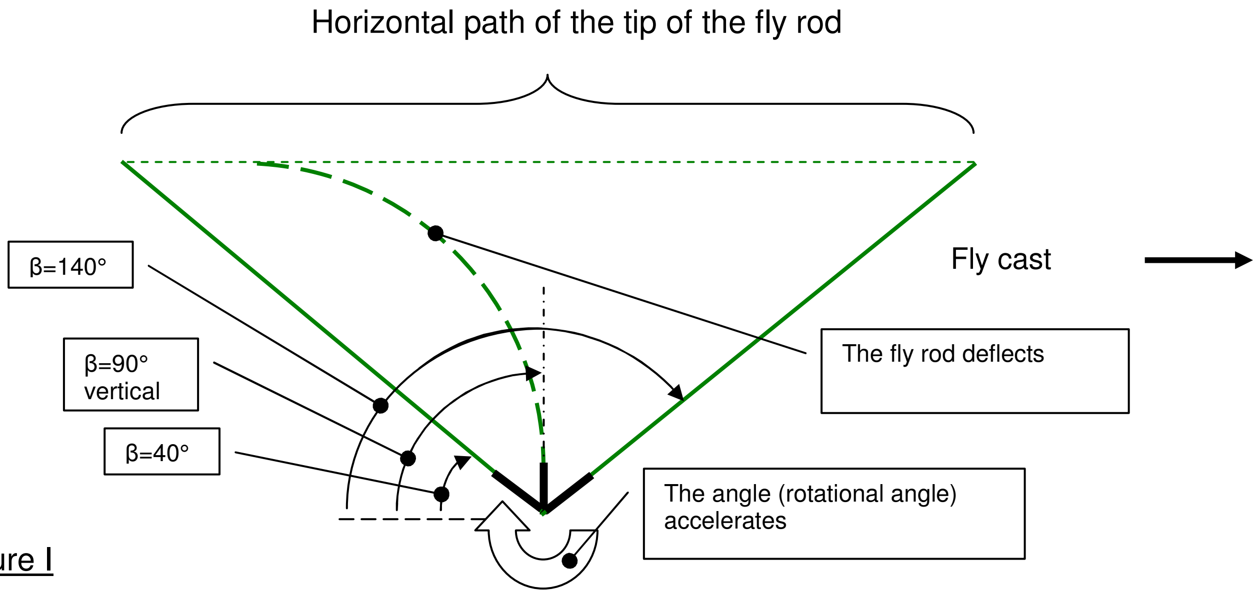


Figure I

[6] For example nor differential neither integral calculus is used. The values are calculated for the corresponding momentary positions of the fly rod and represent values that act in the fly rod at this time point exactly. Effects, that follow due to the dynamical interaction of the forces during the movement are included in the experimental data and therefore it is respected in the calculation that is based on the experiment. For that reason it can be assumed that this approach delivers realistic values.

[7] Single pictures of videos of other fly cast sequences show a nearly identical flection of the fly rod – although when casting the fly rod faster (SAGE TCR) or slowlier. The difference mainly results in a different final velocity of the rod tip.

The common fly rod deflects. The experimental investigations show that the path of the tip of the fly rod is well described by a straight line (see Figure I).

<u>1. Conclusion:</u> The deflection of the fly rod leads to a dynamics of the rod tip moving along a straight line.

B1.1) properties of the deflection of the fly rod

The position of the rod tip has to be determined for the following investigations. The accurate analysis of the deflection of the fly rod is necessary for that purpose. My fly cast sequence sheds light onto this question:

<u>Deflection in the 90° position (vertical position)</u>

To determine the geometry of the flection of the fly rod especially two pictures of my sequence are analyzed in more detail:

These two pictures of the forward and backward fly cast show that the deflection of the fly rod is well described by a segment of a circle. The pictures also show that the length of the fly rod is approximately a quarter of the circumference, when the grip is hold in the 90° position approximately. For the length of the fly rod and the circumference we get:

$$U = \pi * d = 2 * \pi * r; L \cong \frac{1}{4} * U$$

with U= circumference; d = diameter of the circle; r = radius of the circle; L = length of the fly rod

As the length L of the fly rod measures about one quarter of the circumfence U in the vertical position 90° of the grip the radius calculates to:

$$U = 4 * L = 2 * \pi * r \rightarrow r = \mathbf{0.64 * L}$$

The deflection shortens the projection of the fly rod onto the vertical 90° position by

$$L - r = L – 0,64 * L = \mathbf{0.36 * L}$$

Figure II shows the geometrical relations of the fly rod deflection.

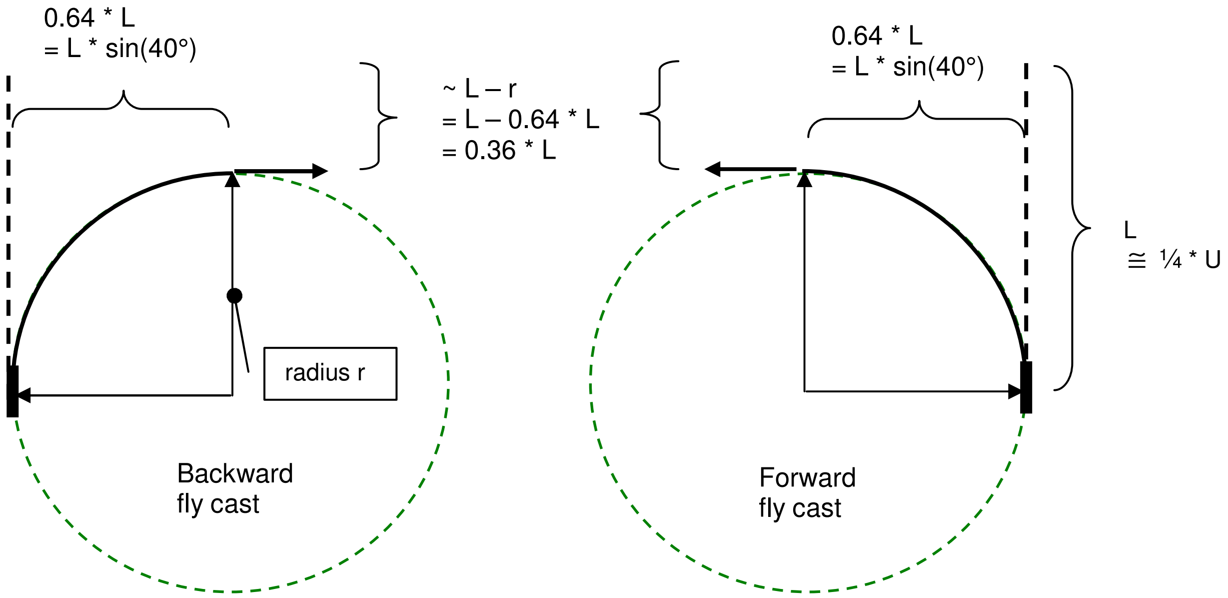


Figure II

***2. Conclusion:* The deflection of the fly rod leads to a shortened projection onto the vertical 90° position by about 1/3 (0.36*L).**

Deflection in the 140° position (final position)

Again two single pictures of the fly cast sequence are analyzed to determine the geometry of the deflection of the fly rod.

The deflection of the fly rod in the 140° position is more complex than in the 90° position. The deflection approximates the form of the segment of an ellipse and cannot be described by a simple geometry. In the vertical 90° position the deflection was distributed equally over the whole length but in the 140° position the (upper) middle-section deflects much stronger than the other sections. It becomes visible that the tip of the fly rod

- continuously follows a straight path

- Is localized approximately over the middle axis

when the grip of the fly cast reached the final position (140° position)

B2) The absolutely stiff fly rod

The following geometrical relations result for the absolutely stiff fly rod:

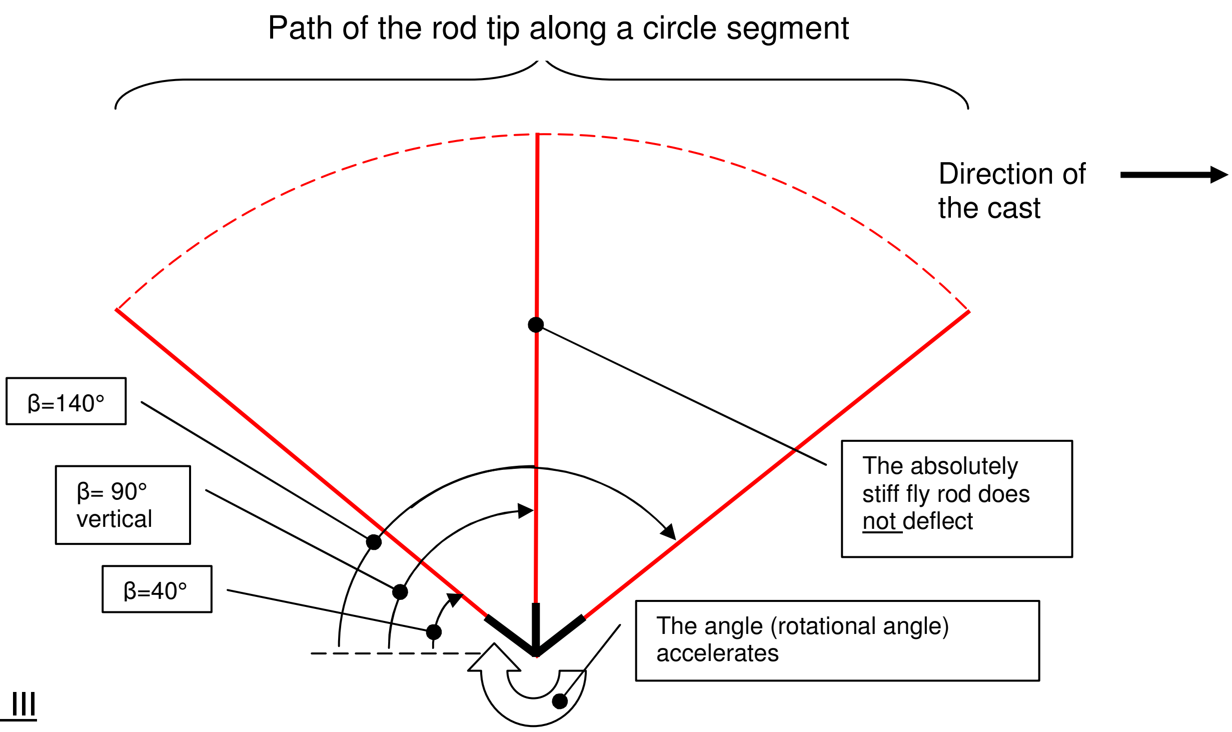


Figure III

The absolutely stiff fly rod does not deflect – even when force is applied. The path of the tip of the rod therefore describes the form of a convex circle segment (see Figure III)[8].

B3) Path of the tip of the fly rod

The whole rotational angle α during the fly cast of both fly rods (flexible/ absolutely stiff) measures to:

$$\alpha = 140° - 40° = \mathbf{100°}$$

The length of the path of the flexible rod´s tip therefore calculates to

$$WRs(f) = 2 * L * \cos(40°) = 2 * 0{,}77 * L = \mathbf{1.54 * L}$$

[8] It is also possible to cast the absolutely stiff fly rod during a small rotational angle in such way that the tip describes a straight line for a short distance. To gain comparable values it is assumed here that the tip of the absolutely stiff fly rod describes a real circle segment.

The path of the tip of the absolutely stiff fly rod describes a circle segment. Its circumfence calculates according to the following formula:

$$WRs(s) = L * \pi * \frac{\alpha(\text{deg})}{180°} = L * \pi * 0.555 = \mathbf{1.75 * L}$$

Subsuming the previous investigations the following Figure compares the geometrical relations of the flexible and the absolutely stiff fly rod: (see Figure IV):

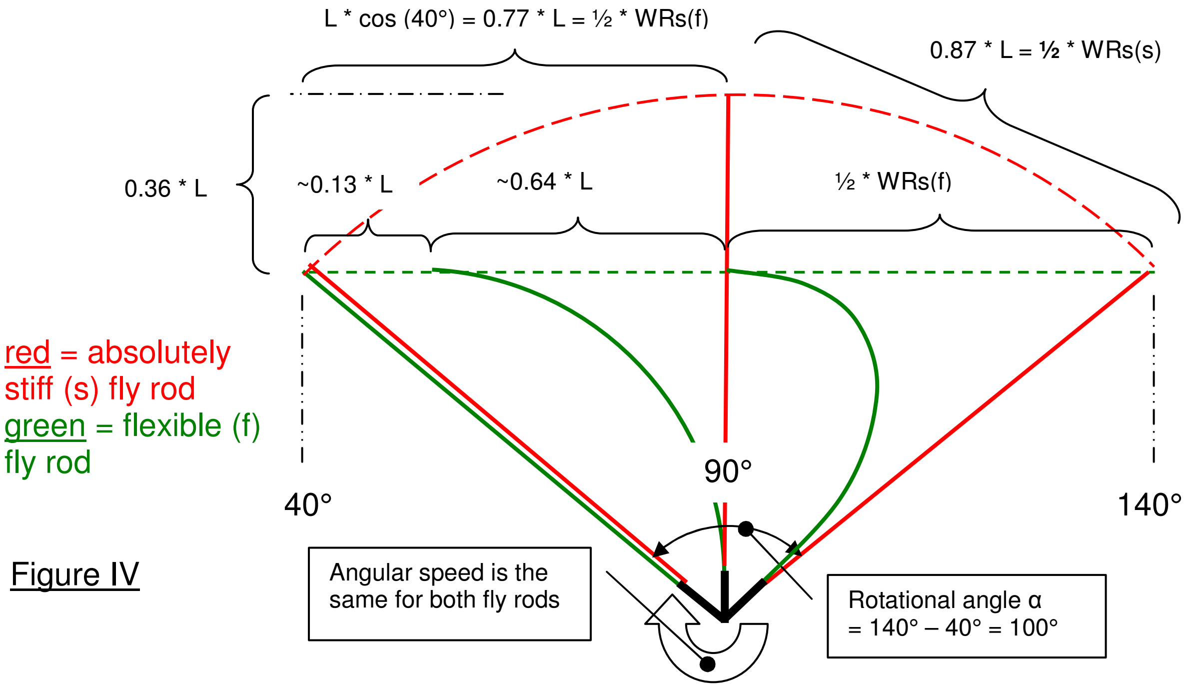


Figure IV

Position rotational angle / position of the rod grip	**40°**	**90° (vertical)**	**140° (end of rotation)**	**sum of the path length (without retraction / discharge)**
Propagated path length of the tip of the flexible fly rod	**0.0**	**(0.13 - 0) * L = 0.13 * L**	**(0.77 - 0.13) * L = 0.64 * L**	**(0.13 + 0.64) * L = 0.77 * L= ½ WRs(f)**
Propagated path length of the tip of the absolutely stiff fly rod	**0.0**	**(0.87 - 0) * L = 0.87 * L = ½ WRs(s)**	**(1.75 - 0.87) * L = 0.87 * L = ½ WRs(s)**	**(0.87 + 0.87) * L =1.75 * L= WRs(s)**

B4) Impact directions

The rotational speed changing the rotational angle α, is accelerated in a similar way for both, the flexile and the absolutely stiff fly rod. But the introduced angular velocity is transferred differently to the tips.

B4.1) Direction of the velocity

Due to the deflection the tip of the flexible fly rod points in the same direction like the elongated fly line during a long part of the path at the beginning of the fly cast. Therefore the fly line is predominantly accelerated horizontally. In marked contrast the tip of the absolutely stiff fly rod does not point in the direction of the accelerated fly line at any time because the tip accelerates the fly line tangentially (see Figure V).

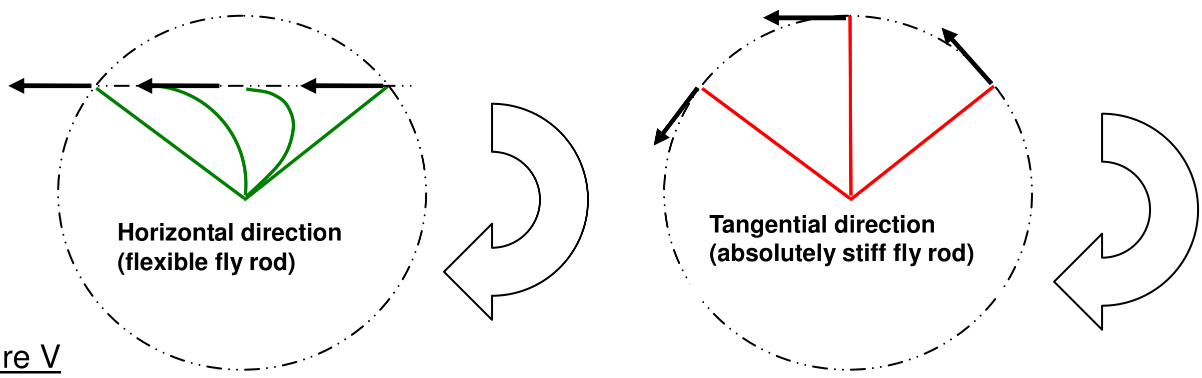


Figure V

***3. Conclusion:* The tip of the flexible fly rod points into the direction of the elongated fly line for a huge part of the path; it accelerates the fly line in the horizontal direction. The tip of the absolutely stiff fly rod does not point into the direction of the elongated fly line at any time; it accelerates the fly line in the tangential direction along the segment of a circle.**

B4.2) Vectorial description of the impact directions and lever arm

As the tips of both fly rods transfer the introduced rotational velocity differently to the fly line, there follow different directions of the impact.

To be able to compare the flexible and the absolutely stiff fly rod, both rods do not only need to have the same rotational speed, but the tips have to cast the fly lines into the same direction. Here the initially formulated constraint shall be concretized.

The fly rod shall be cast into horizontal direction so that the horizontal part of the velocity is in favor. The relations between the fractions and directions (vectors) of the velocity are shown in the following Figure VI:

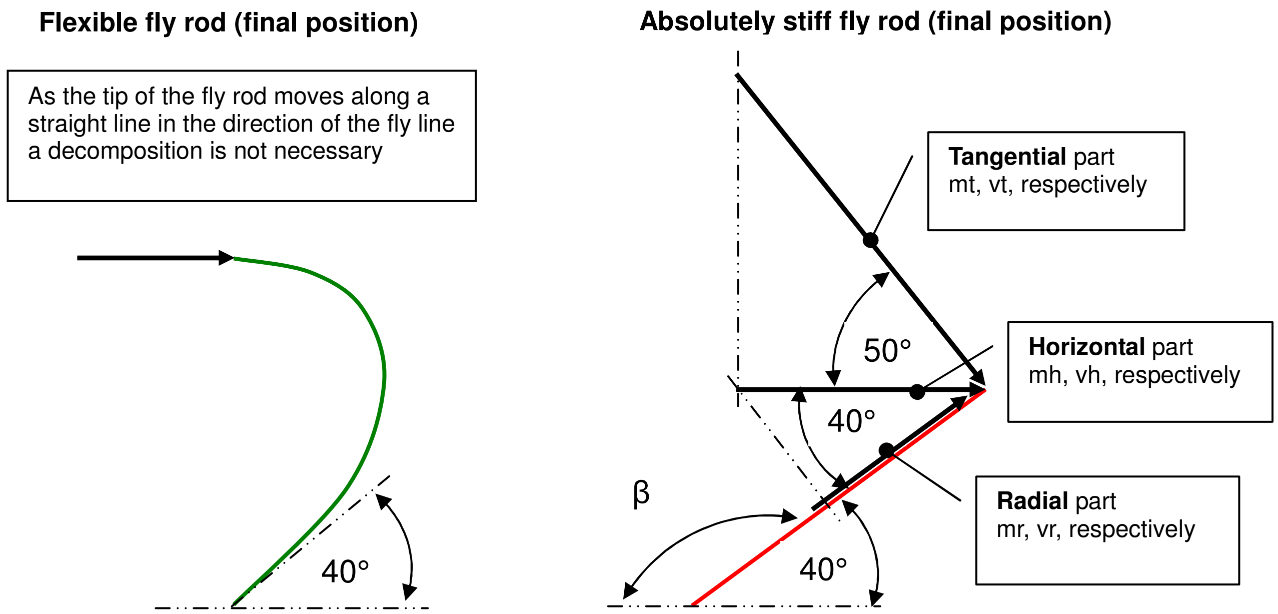


Figure VI

The relation between the tangential and the horizontal velocity of the absolutely stiff fly rod describes as follows:

$$vh = \sin \beta * vt$$

In the final position (140°- position) the horizontal fraction is:

$$vh(140) = \cos(50) * vt = \sin(140) * vt$$

The previous vectorial relation shows, that the velocity components in both directions are correlated[9]. In the final position (140° position) the horizontal fraction calculates to

$$vh(140) = \sin(140) * vt = \mathbf{0.643 * vt}$$

***4. Conclusion:* After a rotational angle α of 100° the horizontal part of the velocity of the tip reduces by a factor of 0.643 for the absolutely stiff fly rod in the final position. This factor reduces with rising rotational angle α and therefore the horizontal part of the velocity that can be reached with the absolutely stiff fly rod.**

It is an advantage if the horizontal part of the velocity is as big as possible. This is only possible for the absolutely stiff fly rod if the rotational angle α is as small as possible.

If the rotational angle α changes, the tip of the fly rod moves the fly line (mass) leading to a torque at the grip. This torque can be calculated with the formula

torque = force * lever arm

[9] And the horizontal part of the velocity can never become larger than the tangential part.

The length of the lever arm is approximately constant for the flexible fly rod during the whole path of the tip and corresponds to the radius of the circle segment r = Hf = 0.64 * L (see section B1.1). For the absolutely stiff fly rod the length of the lever arm changes during the pathway of the tip and corresponds to the length of the rod L in the 90° position.

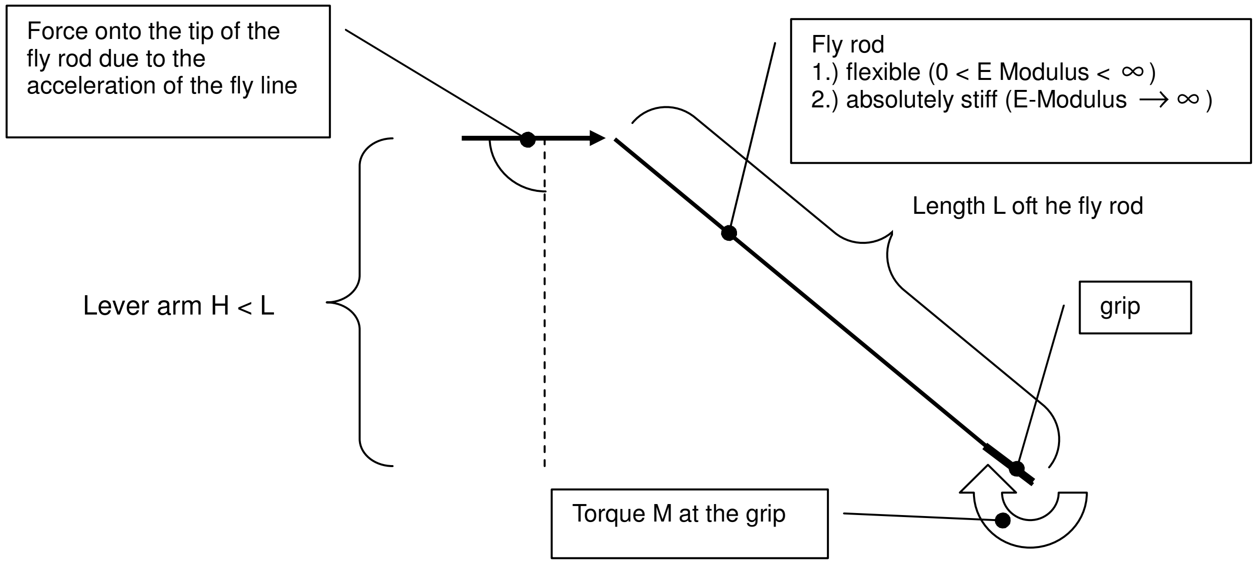


Figure VI

The torques are calculated later when the force that has to be spent by the fly caster is calculated.

C) Dynamical investigations

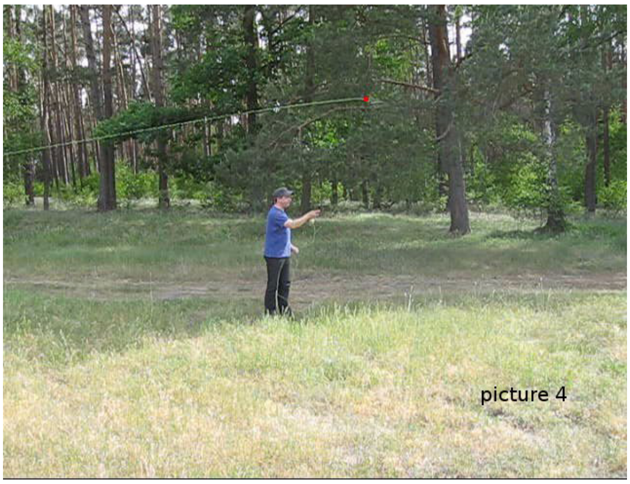
In this section I investigate the dynamics (time dependent relations) in more detail, describing the flexible and the absolutely stiff fly rod during the fly cast.

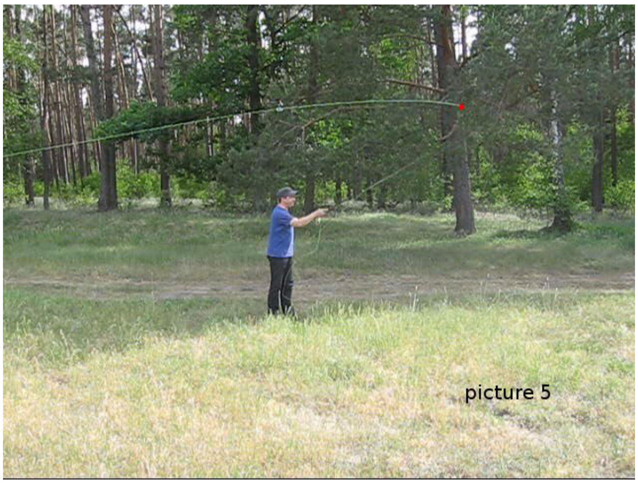
C1) Initiation of the stop, retraction / discharge, respectively

In contrast to the absolutely stiff fly rod the deflection of the flexible fly rod additionally influences the process of the fly cast. The single images of my fly cast sequence show the following typical signs for the initiation of a stop and the retraction/discharge, respectively:

- The linear part of the lower, strong section near the grip increases. The deflecting part of the fly rod decreases and moves into the direction of the tip of the fly rod.
- The hand at the grip of the fly rod has finished the rotation and just follows somehow into the direction of the fly cast

If the single pictures of my fly cast sequence are analyzed according to the aforementioned indications, then the retraction / discharge and the stop of the fly rod, respectively, initiate, when the tip has finished the first half of its path (see picture 3).
The following five single pictures (one picture each 1/30 second) show the forward fly cast around the time point of the stop initiation:





The same can be assumed for my backwards fly cast, because the sequence of the fly cast and the properties of the deflection are virtually identically in both directions.

***5. Conclusion:* The stop, the retraction / discharge, respectively, of the flexible fly rod starts, when its tip has completed about the first half (~50 %) of the path length (rotational path). Then the hand on the grip has reached the final position at 140°[10].**

The following Figure VII shows the relation between stop, the retraction/discharge, respectively, and the position of the tip of the fly rod.

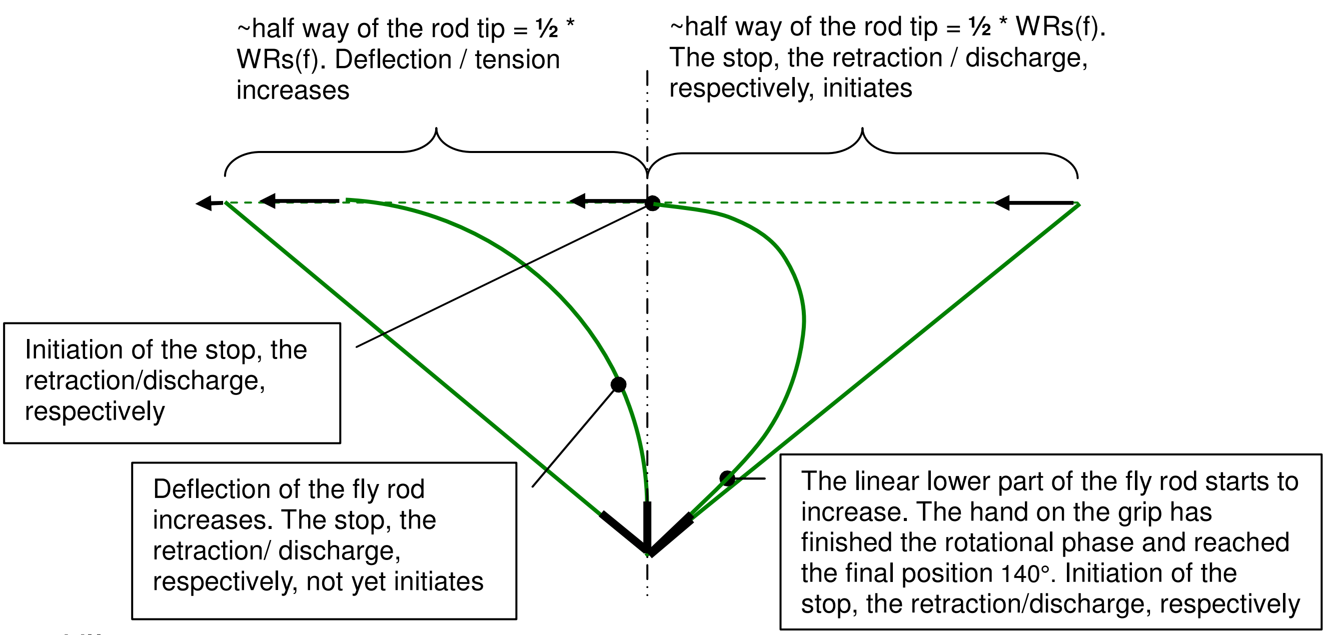


Figure VII

[10] The fact alone, that the retraction/discharge takes half way of the rod tip shows, that it must not be neglected. Independent from the forthcoming conclusions it can be said at this point already that the fly caster has to control not only the increase of tension/deflection but also the release (retraction / discharge) during a long path length of the rod tip.

C2) Path-time-relations from the picture sequence of the fly cast

Now the single positions the rod tip reaches during its path can be attributed to certain time points employing the geometrical relations found in section B) and the picture sequence. For that purpose the picture sequence of my forward fly cast is used in the following. The temporal interval measures 1/30 second. Due to the fact that the fly cast sequence and the deflection of the forward fly cast and the backward fly cast are virtually identical, the determined values also hold for the backward fly cast in a similar way.

pictures 1-3

pictures 4-6

pic 8:~vertical position 90°

pic 10:~initiation of retraction 140°

pictures 11-13

In the last 13th picture the fly cast sequence is terminated already. This can be seen from the following indications:

- In the previous picture 12 the fly rod is already nearly discharged, only the most upper part of the tip is slightly deflected
- The path of the fly rod shows downwards
- The fly rod has just passed the resting position
- The fly line starts to exhibit a noose

Therefore the last picture is neglected for the determination of the duration of the fly cast[11]. The times calculated from the picture sequence are shown in the following table:

Position rotational angle / position of the rod grip	**90°**	**140°**	**Retraction/ discharge /stop until resting position**
Number of pictures until the position of the rotational angle / rod grip is reached	**8**	**10**	**~12**
Overall time (duration)	**8 / 30 = 0.2666s**	**10 / 30 = 0.3333s**	**12 / 30 = 0.40s**
Differences	**40° to 90°**	**90° to 140°**	**from beginning until the end of retraction/discharge**
Time interval Δt (at 30 pictures per second) between the positions	**Δt = (8-0)/30 = 0.2666s**	**Δt = (10-8)/30 = 0.0666s**	**Δt = (12-10)/30 = 0.0666s**

6. conclusion: ***The tips of both rods take different time periods to reach the final horizontal velocity. At the same rotational speed the fly cast of the flexible fly rod takes about 0.0666 seconds longer (the time period of the retraction/discharge) than the fly cast of the absolutely stiff fly rod.***

C3) Determination of velocities from the picture sequence

During the single pictures of the fly cast sequence the tip and the elongated fly line move with a velocity

[11] If the last, 13th picture would also be respected, temporal values would be gathered that do not represent the reality. For example the velocity would decelerate during the retraction/discharge. Then the noose would already form during the retraction/discharge, which is not the case.

$$v = \frac{\Delta path}{\Delta time} = \frac{\Delta p}{\Delta t}$$

The velocities v calculated in such way correspond to the velocities the rod tip reaches between the positions 40° to 90° as well as 90° to 140°. My forward fly cast was exemplarily chosen for the calculations, my backwards fly cast would deliver a comparable picture sequence.

C3.1) Velocities of the tip of the flexible fly rod

In the following table the velocities of the tip of the flexible fly rod are calculated:

Position of the rotational angle / position of the rod grip	**90°**	**140°**	**Retraction/discharge/ stop until resting position**
Covered distance of the rod tip (flexible)	**0.13 * L**	**0.77 * L**	**1.54 * L**
Differences	**40° to 90°**	**90° to 140°**	**from the beginning until the end of retraction/discharge**
Velocity vf of the rod tip ∆w/∆t	**vf = (0.13-0.00)L /0.2666s = 0.487L/s**	**vf = (0.77-0.13)L /0,0666s = 9.61L/s**	**vf = (1.54-0.77)L /0.0666s = 11.56L/s**

C3.2) Velocities of the tip of the absolutely stiff fly rod

For the same picture sequence the velocities of the rod tip of the absolutely stiff fly rod are calculated in the following table[12]. The horizontal part vs,h follows according to section B4.2)

Position of the rotational angle / position of the rod grip	**90°**	**140°**	**Retraction/discharge/ stop until resting position**
Covered distance of the rod tip (absolutely stiff)	**0.87 * L**	**1.75 * L**	**not the case**
Differences	**40° to 90°**	**90° to 140°**	**From beginning until the end of retraction/discharge**
<u>Tangential</u> velocities vs,t of the rod tip	**vs,t = (0.87-0,00)L /0.2666s = 3.26L/s**	**vs,t = (1.75-0.87)L /0.0666s=13.063L/s**	**not the case**
<u>Horizontal</u> velocities vs,h of the rod tip	**vs,h=vs,t*sin(90°) =3.26L/s * 1.00 = 3.26L/s**	**vs,h=vs,t*sin(140°) = 13.063L/s * 0.643 = 8.40L/s**	**not the case**

[12] By using the same picture sequence it is guaranteed that both rods are analyzed with the same rotational speed.

The following Figure VIII shows the graph of the velocities of the tips of both fly rods[13].

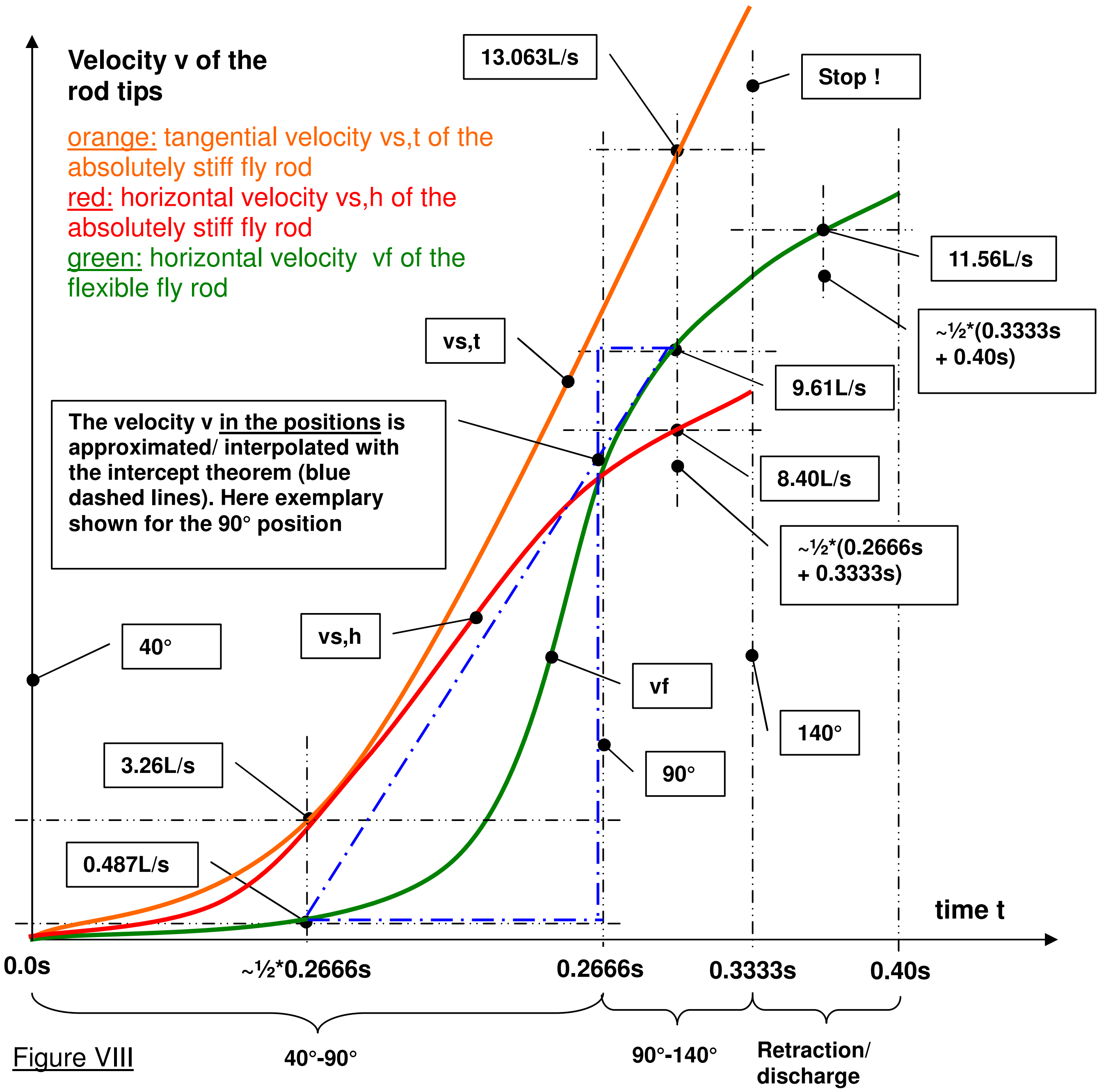


Figure VIII

[13] The average velocities between the positions 40° to 90° and 90° to 140° are attributed to the middle time points for the subsequent investigations, that means: v(40°-90°) ≈ ½ * (0.2666s+0.00s), v(90°-140°) ≈ ½ * (0.2666s+0.3333s) und v(deflection) ≈ ½ * (0.3333s+0.40s). The determined time course of the rotational speed of my casting sequence deviates mainly initially from the linear correlation. Therefore the temporal assignment is not exact. The previously mentioned assignment is chosen for the sake of simplicity, because a more exact determination would be complex and not in proportion to the expected deviation, which is assumed to be neglect able for the undergone investigations – also because it annihilates most probable when comparing both fly rods.

The velocities directly in the positions 90°, 140° and at the end of the retraction / discharge are well approximated by the intercept theorem[14] in the following.

For the tip of the flexible fly rod the horizontal velocity vf calculates to:

$$\frac{0.5*((0.2666-0)+(0.3333-0.2666))}{(9.61-0.487)} = \frac{0.5*0.2666}{vf(90°)-0.487}$$

$$\rightarrow \text{vf}(90°) = \frac{0.5*0.2666*9.123}{0.1666} + 0.487 = 7.30 + 0.487 = \mathbf{7.787}\ \text{L/s}$$

$$\frac{0.5*((0.3333-0.2666)+(0.40-0.3333))}{(11.56-9.61)} = \frac{0.5*(0.3333-0.2666)}{vf(140°)-9.61}$$

$$\rightarrow \text{vf}(140°) = \frac{0.5*0.0666*1.95}{0.0666} + 9.61 = 0.975 + 9.61 = \mathbf{10.585}\ \text{L/s}$$

$$\frac{0.5*(0.40-0.3333)}{(11.56-10.585)} = \frac{0.5*(0.40-0.3333)}{vf(end)-11.56}$$

$$\rightarrow \text{vf(end)} = \frac{0.5*0.0666*0.975}{0.0333} + 11.56 = 0.975 + 11.56 = \mathbf{12.535}\ \text{L/s}$$

For the tip of the absolutely stiff fly rod the horizontal velocity vs,h calculates to:

$$\frac{0.5*((0.2666-0)+(0.3333-0.2666))}{(8.40-3.26)} = \frac{0.5*0.2666}{vs,h(90°)-3.26}$$

$$\rightarrow \text{vs,h}(90°) = \frac{0.5*0.2666*5.14}{0.1666} + 3.26 = 4.113 + 3.26 = \mathbf{7.373}\ \text{L/s}$$

$$\frac{0.5*((0.3333-0.2666)+(0.40-0.3333))}{(8.40-7.373)} = \frac{0.5*(0.3333-0.2666)}{vs,h(140°)-8.40}$$

$$\rightarrow \text{vs,h}(140°) = \frac{0.5*0.0666*1.027}{0.0333} + 8.40 = 1.027 + 8.40 = \mathbf{9.427}\ \text{L/s} = \text{vs,h(end)}$$

[14] Intercept theorem: http://en.wikipedia.org/wiki/Intercept_theorem. For the example 90° the equations according to the intercept theorem denote to:
(½ t(40° to 90°)+½ t(90° to 140°)) : (v(90° to 140°)- v(40° to 90°)) = ½ t(40° to 90°) : (**v(90°)**-v(40° to 90°))

The comparison of the final velocities v(end) and the initial velocities v(40°-90°) of the tips of both fly rods results in:

vf(end) / vs,h(end) = 12.53 / 9.43 = **1.33**

vf(40°-90°) / vs,h(40°-90°) ≈ 0.487 / 3.26 = **0.15**

<u>7. Conclusion:</u> The tip of the flexible fly rod has a final horizontal velocity which is about 33 % higher in comparison to the absolutely stiff fly rod. In marked contrast the tip of the flexible fly rod has only 15 % of the initial velocity in comparison to the absolutely stiff fly rod at the beginning of the fly cast (initial velocity)[15].

The comparison of the velocities of the tips of both fly rods in between the positions 40° to 90° and 90° to 140° results in:

vf(40°-90°) / vf(90°-140°) = 0.487 / 9.61 = **0.051**

vs(40°-90°) / vs(90°-140°) = 3.26 / 8.40 = **0.388**

The comparison of the velocities of the tip of the flexible fly rod at the beginning and at the end of the stop, the retraction / discharge, respectively delivers:

vf(140°) / vf(end) = 10.585 / 12.535 = **0.84**

<u>8. Conclusion:</u> At the beginning of the cast the tip of the flexible fly rod has 5 % (0.051) and the one of the absolutely stiff fly rod 39 % (0.388) of the velocity which they reach at the end of the cast. The tip of the flexible fly rod has more than 80 % (0.84) of its final velocity at the beginning of the stop / retraction / discharge[16].

C4) Calculation of the accelerations

The acceleration is defined as the change of the velocity in time[17]. Now the acceleration can be calculated for each section from the previously calculated velocities:

[15] The introduced rotational speed is transferred into the velocity of the tip with retardation. The rotational speed that is spent at the beginning of the fly cast nearly completely leads to deflection of the fly rod (which causes a remarkable increase of the potential tension force) and not to increasing velocity of the tip. When the tip reaches approximately the vertical 90° position, the tip of the flexible fly rod has reached the horizontal velocity of the tip of the absolutely stiff fly rod and continues to accelerate. Finally the tip of the flexible fly rod exhibits a much larger horizontal velocity than the tip of the absolutely stiff fly rod at the end of the fly cast.

[16] The fact that the tip of the flexible fly rod nearly reaches its final velocity at the initiation of the stop / the retraction / discharge is often used as an argument, to underestimate the meaning of its deflection. This view neglects that exactly due to the deflection the tip of the flexible fly rod reaches a remarkable higher final horizontal speed than the tip of the absolutely stiff fly rod and that the flexible fly rod distributes the velocity of the tip differently along its path (see 7th and 8th conclusion). The different distribution of the velocity of the rod tip will be analyzed in more detail when the efficiency is considered.

[17] The acceleration is the derivative of the velocity: a = v'.

$$a = \frac{\Delta velocity}{\Delta time} = \frac{\Delta v}{\Delta t}$$

For the tip of the flexible fly rod the acceleration af calculates to:

$$af1 = a(40°\text{-}90°) = \frac{0.487-0}{0.5*(0.2666-0)}\ \frac{L}{s^2} = \frac{0.487}{0.1333} = \mathbf{3.65}\frac{L}{s^2}$$

$$af2 = a(90°) = \frac{9.61-0.487}{0.5*((0.2666+0.3333)-0.2666)}\ \frac{L}{s^2} = \frac{9.123}{0.1666} = \mathbf{54.76}\frac{L}{s^2}$$

$$af3 = a(90°\text{-}140°) = \frac{10.585-7.787}{0.3333-0.2666}\ \frac{L}{s^2} = \frac{2.796}{0.0666} = \mathbf{41.98}\frac{L}{s^2}$$

$$af4 = a(140°) = \frac{11.56-9.61}{0.5*((0.3333+0.40)-(0.2666+0.3333))}\ \frac{L}{s^2} = \frac{1.95}{0.0666} = \mathbf{29.23}\frac{L}{s^2}$$

$$af5 = a(discharge) = \frac{12.535-10.585}{0.40-0.3333}\ \frac{L}{s^2} = \frac{1.95}{0.0666} = \mathbf{29.23}\frac{L}{s^2}$$

For the tip of the absolutely stiff fly rod the acceleration[18] as calculates to:

$$as1 = a(40°\text{-}90°) = \frac{3.26-0}{0.5*(0.2666-0)}\ \frac{L}{s^2} = \frac{3.26}{0.1333} = \mathbf{24.45}\frac{L}{s^2}$$

$$as2 = a(90°) = \frac{8.40-3.26}{0.5*((0.2666+0.3333)-0.2666)}\ \frac{L}{s^2} = \frac{5.14}{0.1666} = \mathbf{30.85}\frac{L}{s^2}$$

$$as3 = a(90°\text{-}140°) = \frac{9.427-7.373}{0.3333-0.2666}\ \frac{L}{s^2} = \frac{2.054}{0.0666} = \mathbf{30.79}\frac{L}{s^2}$$

The following Figure IX shows the curves of the accelerations of the tips of both rods:

[18] In the following the index „h“ for „horizontal“ is omitted. All accelerations refer to the horizontal direction.

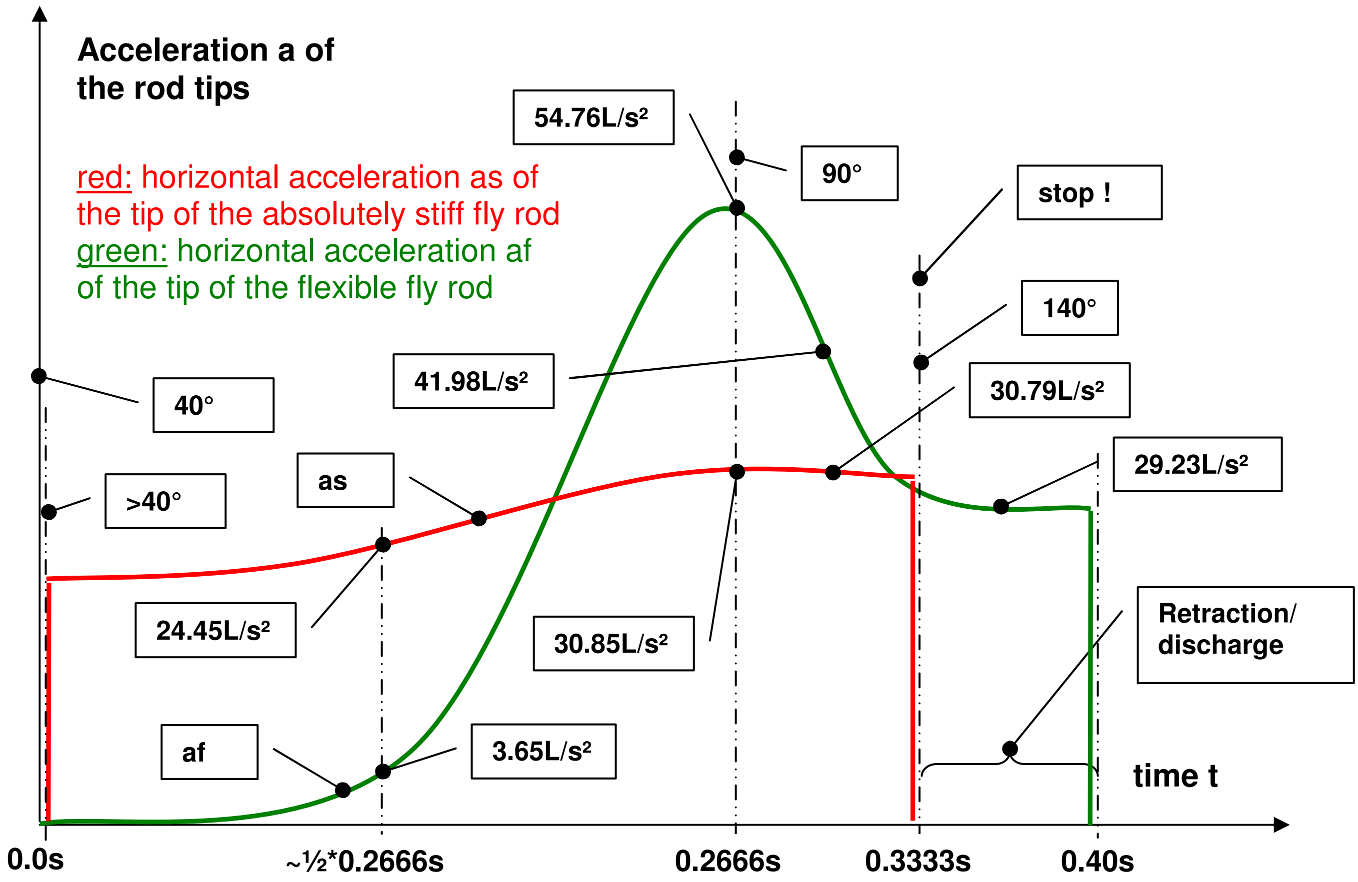


Figure IX

D) Calculation of the forces

In this section I investigate the forces in more detail, which are present during the fly cast of the flexible and the absolutely stiff fly rod: For the calculation of the forces Newton´s second law is used:

Force = mass * acceleration; F = m * a

The mass m is represented by the weight of the elongated fly line which pulls at the tip of the fly rod[19].

[19] The calculation of forces according to Newton is done with the mass that has to be accelerated. The assumption of massless fly rods do not comprise forces necessary to rotate the fly rod mass itself in the calculations. As the fly rods have significantly more mass than the fly line this fraction of neglected forces is high. It is nevertheless possible to refer to Newton´s second law of motion in the current situation as the deflection should influence the acceleration of both masses, the rods masses (flexible / absolutely stiff) and the mass of the fly line, in a similar way. My calculation in annex 2 shows that this assumption for the influence of the mass is correct.

D1) The potential force of tension that acts on the tip

The tension on the fly line (mass m) and the rotation of the own mass of the fly rod leads to its deflection. For the further investigations we assume:

- At the averaged time point between the 40° and the 90° position (~65°) the fly rod tip has not moved at all or covered a neglectable distance
- The retraction / discharge occurs from half the way of the fly rod tip

Both assumptions follow from the picture sequence (see section C2). The assumptions lead to the following displacements of the tip of the flexible fly rod (see table):

Position of the rotational angle / position of the rod grip	**40° to 90° ~½ of time**	**90°**	**140°**	**~½ time retraction / discharge, stop, respectively**	**~ full time retraction / discharge, stop, respectively**
Covered distance of the rod tip (WRs) <u>with</u> deflection	**~0.0L**	**0.13L**	**0.77L**	**~(0.77+0.5*0.77)*L = 1.155L**	**1.54 L**
Covered distance of the rod tip along the axis of the rod (<u>without</u> deflection)	**~0.5*0.77L = 0.385L**	**0.77 L**	**1.54 L**	**1.54 L**	**1.54 L**
difference = displacement x = deflection	**0.385 L**	**0.64 L**	**0.77 L**	**0.385 L**	**0.0 L**
Difference of the displacement ∆x = change of the deflection	**(0.385-0)*L = <u>0.385 L</u>**	**(0.64-0.385)*L = <u>0.255 L</u>**	**(0.77-0.64)*L = <u>0.13 L</u>**	**(0.385-0.77)*L = -<u>0.385 L</u>**	**(0-0.385)*L = -<u>0.385 L</u>**

<u>9. Conclusion:</u> The displacement measures the deflection of the fly rod. The displacement increases for more than until the 90° position is reached. The whole displacement introduced into the rotation (change of rotational angle) increases the deflection of the fly rod.

The whole potential energy that is stored in the fly rod is transferred to kinetic energy via the retraction / discharge due to the deflection[20], because the fly caster does not continue the rotation and therefore does not infer more energy or work, respectively. As the velocities are known at the beginning and the end of the retraction/discharge, it is possible to calculate the potential energy.

$$E_{pot,(discharge)} = \tfrac{1}{2} \cdot m \cdot vf(end)^2 - \tfrac{1}{2} \cdot m \cdot vf(140°)^2$$

$$= \tfrac{1}{2} \cdot m \cdot \left(12.535^2 \frac{L^2}{s^2} - 10.585^2 \frac{L^2}{s^2}\right) = \mathbf{22.54} \frac{m \cdot L^2}{s^2}$$

[20] It is correct to look at the energy E = ½ m v² to treat the problem according to the energy conservation law. The energy, the fly rod releases during retraction / discharge must have been spent by the fly caster before.

To get the potential tension force, the energy must be divided through the translated distance:

Fpot,5 = F(discharge) = Epot,(discharge) / distance = 22.54 / 0.77L
= **29.27** $\frac{m*L}{s^2}$ = Fpot,all

This tension force which acts during the retraction / discharge must have been spent by the fly caster. It can be estimated and approximately distributed by the known displacements x and Δx, respectively

Until the initiation of the retraction / discharge, it follows:

Fpot,1 = 0.385L/0.77L * 29.27 = **14.63** $\frac{m*L}{s^2}$ = Fpot(40°-90°)

ΔFpot,1 = 0.385L/0.77L * 29.27 = **14.63** $\frac{m*L}{s^2}$

Fpot,2 = 0.64L/0.77L * 29.27 = **24.33** $\frac{m*L}{s^2}$ = Fpot(90°)

ΔFpot,2 = 0.255L/0.77L * 29.27 = **9.69** $\frac{m*L}{s^2}$

Fpot,3 = 0.77L/0.77L * 29.27 = **29.27** $\frac{m*L}{s^2}$ = Fpot(90°-140°)

ΔFpot,3 = 0.13L/0.77L * 29.27 = **4.94** $\frac{m*L}{s^2}$

Fpot,4 = 0.77L/0.77L * 29.27 = **29.27** $\frac{m*L}{s^2}$ = Fpot(140°)

ΔFpot,4 ≈ 0.00L/0.77L * 29.27 ≈ **0.00** $\frac{m*L}{s^2}$

With the initiation of the retraction / discharge it follows[21]:

Fpot(discharge,initial) = 0.77L/-0.77L * 29.27 = **-29.27** $\frac{m*L}{s^2}$

ΔFpot(discharge,initial) ≈ 0.00L/-0.77L * 29.27 = **0.00** $\frac{m*L}{s^2}$ ≈ ΔFpot(140°)

Fpot(discharge,mid) = 0.385L/-0.77L * 29.27 = **-14.63** $\frac{m*L}{s^2}$

ΔFpot(discharge,mid) = -0.385L/-0.77L * 29.27 = **14.63** $\frac{m*L}{s^2}$

Fpot(discharge,end) = 0.00L/-0.77L * 29.27 = **0.00** $\frac{m*L}{s^2}$

[21] The direction of the retraction / discharge leads to a negative sign. Like for the elastic spring the path length which leads to the deflection (charging) and retraction (discharching) must have different signs.

$$\Delta\text{Fpot,end} = -0.385L/-0.77L * 29.27 = \mathbf{14.63}\ \frac{m*L}{s^2} = \Delta\text{F(discharge,end)}$$

The following Figure X shows the curve of the potential tension force of the flexible fly rod[22].

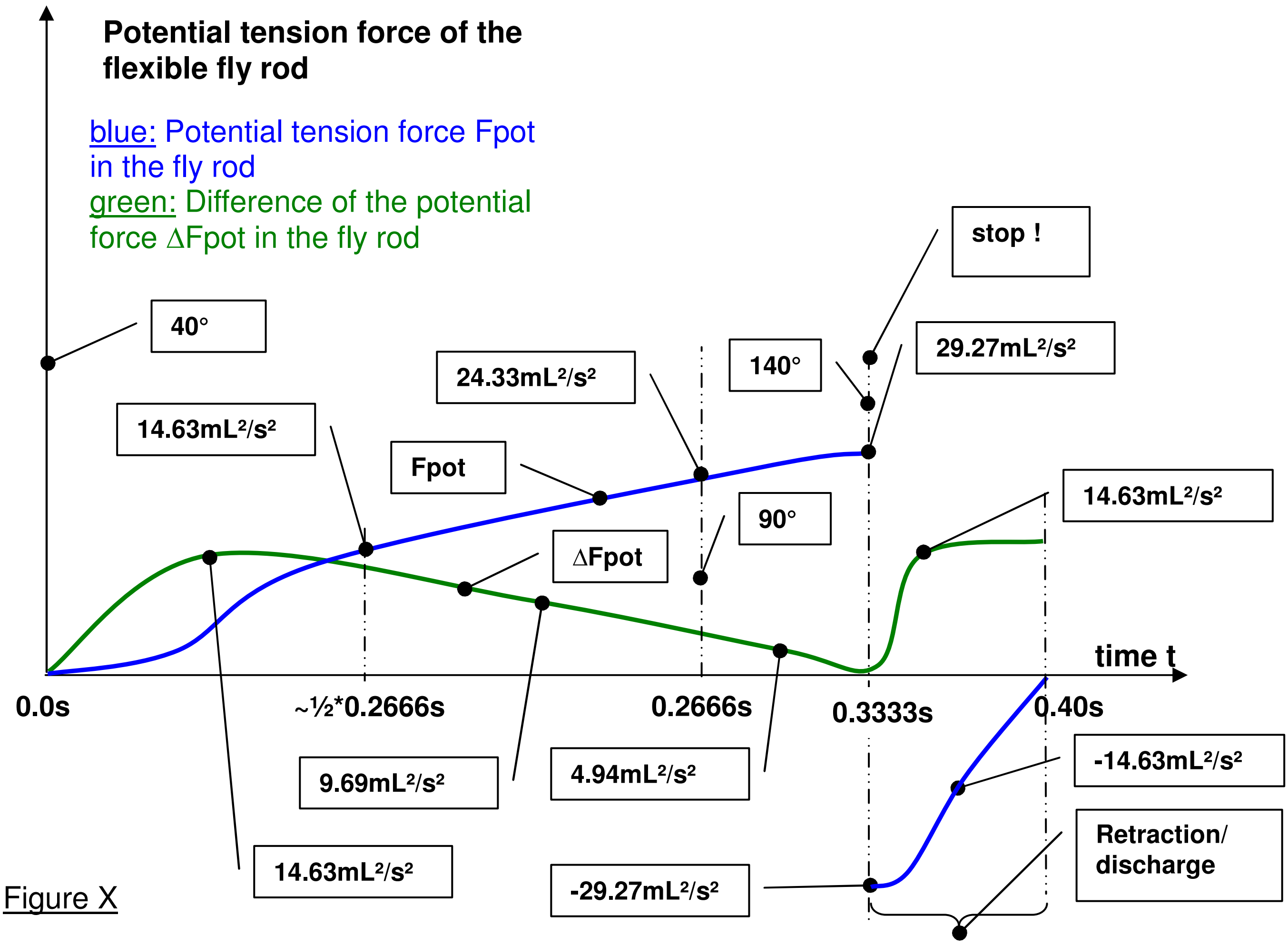


Figure X

The calculated course of the potential tension force shows a continuous change (the current results clearly point it out including the picture sequence in section C2). The continuous change of the potential tension force leads to an optimized transfer of velocity onto the fly line. If the fly caster neglects this fact (and settles pressure points - „handshock“), the tip of the fly rod becomes uneasily and introduces waves as well as

[22] According to the initially stated constraints it is assumed that both fly rods do not carry mass. The fact that the deflection of the fly rod increases in spite of decreasing acceleration has to be attributed to the real existing mass of the rod. The approximately stated assignment of the potential tension force as well as the drawn conclusions are still valid, because the own mass leads to increasing forces onto the rod tip and therefore also the acceleration and the spent force equally increase. The experimental data uniquely show that the deflection of the fly rod increases until the stop, the initiation of the retraction/discharge, respectively ! For the calculation of the potential tension forces and the efficiency this effect can be taken into account in future work.

tailing loops[23] into the fly line. This leads to a reasonable loss of the transferred energy over the path length of the tension of the fly line. The continuous change of the tension force during the duration of the fly cast helps to its controlled formation and depletion guiding the rod tip to run smoothly[24].

D2) Forces acting on the tips of both fly rods

For the flexible fly rod also kinetic forces (acceleration forces) act next to the potential tension force (force of position). For the absolutely stiff fly rod only the kinetic force acts, because the rod does not deflect and therefore potential tension force cannot be stored.

The fact that the absolutely stiff fly rod cannot take up potential tension force leads to a different distribution of the forces in comparison to the flexible fly rod.

***10. Conclusion:* The initial rotation leads to increasing deflection for the flexible fly rod causing an increase of the potential tension force without an acceleration of the mass of the fly line. For the absolutely stiff fly rod each initial movement immediately leads to an acceleration of the fly line and the immediate need to spend force for it.**

For the flexible fly rod it follows:

$F = m * a + \Delta Fpot$[25]

$Ff0 \approx 0 = F(>40°)$ (see 10. Conclusion)

$$Ff1 = m * 3.653 \frac{L}{s^2} + 14.63 \frac{m*L}{s^2} = \mathbf{18.28} \frac{m*L}{s^2} = Ff(40°\text{-}90°)$$

$$Ff2 = m * 54.76 \frac{L}{s^2} + 9.69 \frac{m*L}{s^2} = \mathbf{64.45} \frac{m*L}{s^2} = Ff(90°)$$

[23] This footnote can be dropped in the english version, because 'tailing loop' is an english expression.
[24] For a comparison of the controlled generation and release of potential tension forces the wings of an arch can be considered, because the potential tension forces should develop similarly to that case (there are differences of course, e.g. the fact that the fly rod releases the potential tension force starting from the initiation of the stop – the archer might retard this; another fact might be that in contrast to the wings of an arch the tip of the fly rod has already a relative speed with the initiation of the retardation / discharge). The archer continuously rises the tension of the arch wings aiming to let them accelerate the arrow with highest precision – also the fly caster should rise the tension of the fly rod as continuously as possible, aiming to let the tip of the fly rod accelerate the fly line precisely. The arrow shall not oscillate during its flight to avoid the loss of energy – also waves in the fly line should be avoided. The fly caster can and should not be investigated equally with an archer. But there are parallel effects regarding the impact of the potential tension force and their controlled generation and release, which can show the influence of the tension better than other comparisons. This comparison has made the impact of the potential tension forces more understandable for Dr. Schmitt (see acknowledgments) as he had personally stated.
[25] The fly caster has to spend the change of the potential tension force ΔFpot. Only the change of a state needs force that performs work. For the sake of simplicity ΔFpot,1 is attributed to the position between 40° and 90° and ΔFpot,2 the position 90°. This assignment is more reliable because by this fact higher forces are assigned to the flexible fly rod in comparison to the alternative assignment by the average.

Ff3 = m * 41.98 $\frac{L}{s^2}$ + 4.94 $\frac{m*L}{s^2}$ = **46.92** $\frac{m*L}{s^2}$ = Ff(90°-140°)

Ff4 = m * 29.27 $\frac{L}{s^2}$ + 0.0 $\frac{m*L}{s^2}$ = **29.27** $\frac{m*L}{s^2}$ = Ff(140°)

Ff5 = **14.63** $\frac{m*L}{s^2}$ = Ff(discharge,end)

For the absolutely stiff fly rod it follows:

F = m * a; ΔFpot = 0;

Fs0 ≈ **24.45** $\frac{m*L}{s^2}$ = Fs(>40°) (see 10. conclusion)

Fs1 = **24.45** $\frac{m*L}{s^2}$ = Fs(40°-90°)

Fs2 = **30.85** $\frac{m*L}{s^2}$ = Fs(90°)

Fs3 = **30.79** $\frac{m*L}{s^2}$ = Fs(90°-140°);

D3) Forces (torques) at the grip of both fly rods

During the rotation of the fly rod a torque M acts on the grip, which has to be spent by the fly caster. The torque calculates as product of force and length of the lever.

The forces F have been calculated in section D2) and the lever in section B4). According to this the length of the lever arm of the flexible fly rod is virtually constant along the whole path of the rod tip. For the absolutely stiff fly rod the length of the lever arm in the 40°, 90° and 140° position is known. On the half way between these positions the length of the lever is approximately the algebraic average[26].

For the flexible fly rod it follows:

Mf0(>40°) ≈ 0

Mf1(40-90°) = 18.28 * 0.64 L = **11.70** $\frac{m*L^2}{s^2}$

Mf2(90°) = 64.45 * 0.64 L = **41.25** $\frac{m*L^2}{s^2}$

[26] Taking a shortened projection of the lever arm into account for the absolutely stiff fly rod leads to a calculation of the "lower limit" of the effort ("best case") because in the following also the diminished horizontal part only of the force is respected (see section B4). This treatment promotes the effort of the absolutely stiff fly rod which would be higher in reality regarding the investigated rotational movement. This conservative treatment also pays attention to the fact that the tip of the absolutely stiff fly rod can be conducted along a straight line at least for a short distance (small rotational angle)

Mf3(90°-140°) = 46.92 * 0.64 L = **30.02** $\frac{m*L^2}{s^2}$

Mf4(140°) = 29.27 * 0.64 L = **18.73** $\frac{m*L^2}{s^2}$

Mf5 = 14.64 * 0.64 L = **9.36** $\frac{m*L^2}{s^2}$ = M(discharge,end)

For the absolutely stiff fly rod it follows:

Ms0(>40°) ≈ 24.45 * 0.64 L = **15.65** $\frac{m*L^2}{s^2}$

Ms1(40°-90°) = 24.45 * ½ (0.64 + 1.0) L = **20.05** $\frac{m*L^2}{s^2}$

Ms2(90°) = 30.85 * 1.0 L = **30.85** $\frac{m*L^2}{s^2}$

Ms3(90°-140°) = 30.79 * ½ (1.0 + 0.64) L = **25.25** $\frac{m*L^2}{s^2}$

Ms4(140°) ≈ 30.79 * 0.64 L = **19.70** $\frac{m*L^2}{s^2}$

***11. Conclusion:* The torque which has to be spent by the fly caster rises with the square of the length L of the fly rod[27].**

[27] I sold my first fly rod measuring 10 feet (3.0 meter) for fly lines class 8 after short usage. It was so stiff that my wrest could not transfer the rotational movement without reasonable effort of force and started to hurt after short time in spite of the fact that this fly rod was "only" 10 % longer than the usual 9 feet (2.70 m). The fact that the force (torque) rises with the square of the length of the fly rod explains the reasonable increase of the additional force effort, which is significantly larger than the 10 % length increase of the fly rod. Since then I chose my few fly rods measuring 10 feet, accurately and paid attention to a „fast deep action". During this action the fly rod deflects comparatively more in the lower segment. Therefore the lever arm shortens on the one hand and the deflection and the path of the retraction / discharge increases. This reduced the effort of force and such a fly rod finally still carries enough potential of retraction, to throw the fly line far. Also this example shows that the deflection of the fly rod plays an important role for the effort of force !

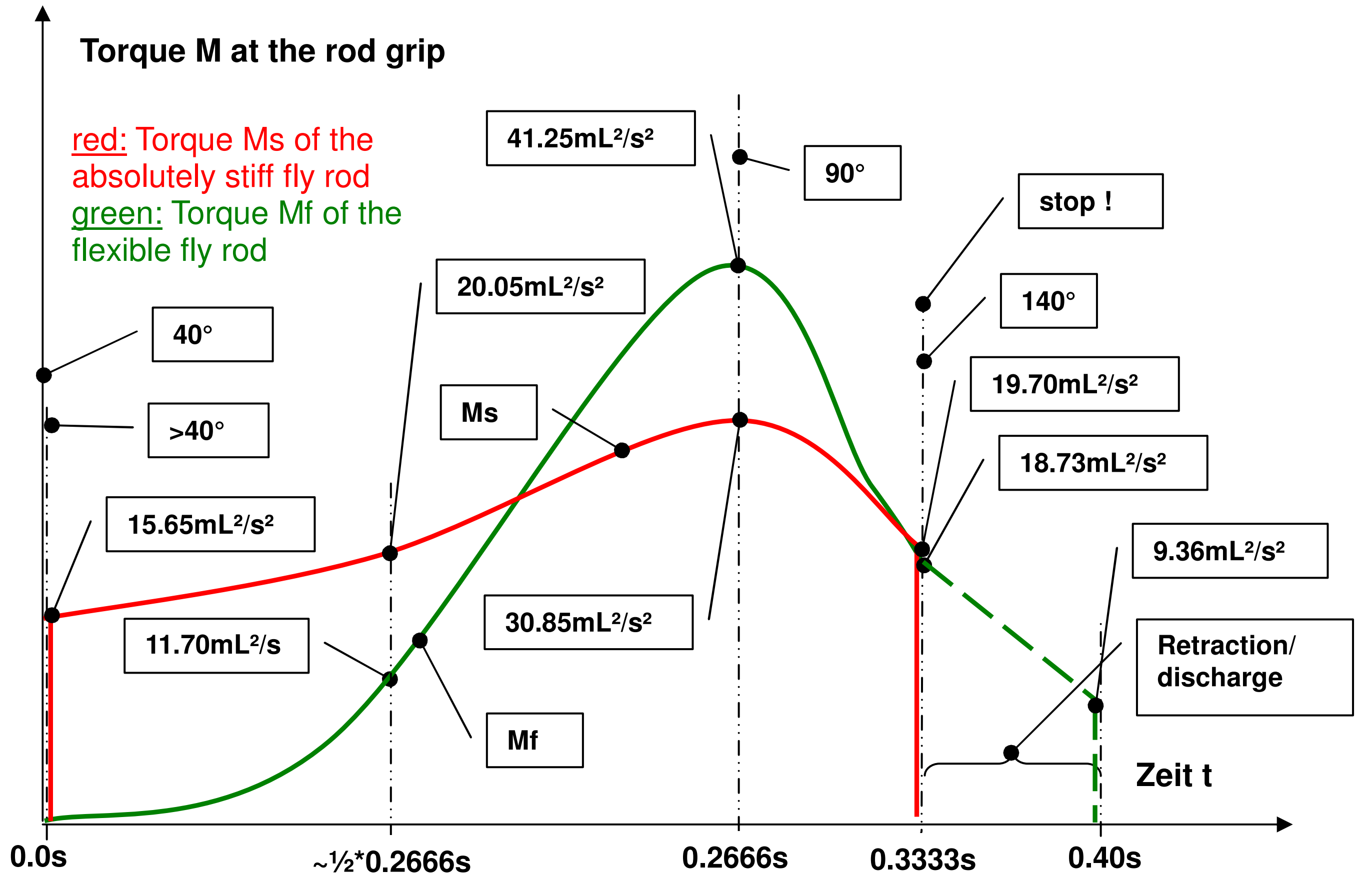


Figure XI

E) Efficiency and transfer of energy, respectively

The ratio of the used energy and the spent energy is called efficiency or effectiveness η, respectively. As bigger the efficiency as better the energy is transferred.

E1) Energy of the fly rod tips (benefit)

The energy E, the tips of the flexible and the absolutely stiff fly rods reach at the end of the fly cast, correspond with the usable energy (benefit)[28].This energy can be calculated by the formula:

$$E = ½ * m * v(end)^2$$

[28] In the following only the energies of the rod tips are respected on the side of the used energy (advantage). It is beyond the scope of this study to analyse how far these energies can be attributed to the fly line. It might be a topic of future works. Surely it also depends on the fly caster´s ability, to transfer the velocity of the rod tips onto the fly line in an optimized fashion. Basically the maximized horizontal velocity at the end delivers the biggest potential for the highest speed of the fly line.

The final velocities vf(end) and vs(end), respectively, of the tips of both fly rods have been calculated in section C3).

For the flexible fly rod it follows:

$$Ef = \frac{1}{2} * m * 12.535^2 \frac{L^2}{s^2} = \mathbf{78.56} \frac{m * L^2}{s^2}$$

For the absolutely stiff fly rod it follows:

$$Es = \frac{1}{2} * m * 9.427^2 \frac{L^2}{s^2} = \mathbf{44.43} \frac{m * L^2}{s^2}$$

E2) Work, energy, respectively of the rotation (effort)

The work which has to be spent by the fly caster for the rotation of the fly rod, corresponds to the effort. It is calculated by the formula

$$A = \sum M * \alpha(rad)$$

The angle α(deg) measures 100° for both rods, corresponding to a radian measure (rad) of

$$\alpha(rad) = \pi * \frac{\alpha(\deg)}{180°} = \pi * 0.555 = \mathbf{1.75}$$

If the grip is in the final position (140° position), then the fly caster has spent the whole work or energy, respectively, for the fly cast. During the retraction / discharge the rotational angle does not change any more and therefore the fly caster does not spend any further work / energy.

<u>12. Conclusion:</u> During the retraction / discharge of the flexible fly rod the fly caster does not spend any more work / energy, because the rotational angle does not change (α=0). During the retraction / discharge only the flexible fly rod is working while increasing the velocity of its tip.

The sum of the torques in the rod grip working during the path length of the rod tip is well approximated by the trapezoidal formula in the following[29].

sum Mf(t) of the torques of the flexible fly rod:

$\sum Mf(t)$ = ½ (M0 + M1) * ½ * 0.2666s ~40° - 65°

+ ½ (M1 + M2) * ½ * 0.2666s ~65° - 90°

[29] More exact trapezoidal rule. http://en.wikipedia.org/wiki/Trapezoidal_rule

+ ½ (M2 + M3) * ½ (0.3333s-0.2666s) ~90° - 115°

+ ½ (M3 + M4) * ½ (0.3333s-0.2666s) ~115° - 140°

+ ½ (M4 + M5) * (0.4s-0.3333s) retraction / discharge

= ½ * 11.70 * 0.1333 + ½ * 52.95 * 0.1333 + ½ * 71.27 * 0.0333

+ ½ * 48.75 * 0.0333 + ½ * 28.09 * 0.0666

This value becomes zero after multiplication with α=0, see 12th conclusion

= 0.779 + 3.529 + 1.186 + 0.81 + 0.00 = **6.30** $\frac{m*L^2}{s^2}$ *s

Af(t) = $\sum Mf(t)$ * α(rad) = 6.30 * 1.75 = **11.025** $\frac{m*L^2}{s^2}$ *s

Sum Ms(t) of the torques of the absolutely stiff fly rod:

$\sum Ms(t)$ = ½ (M0 + M1) * ½ * 0.2666s ~40° - 65°

+ ½ (M1 + M2) * ½ * 0.2666s ~65° - 90°

+ ½ (M2 + M3) * ½ * (0.3333s-0.2666s) ~90° - 115°

+ ½ (M3 + M4) * ½ * (0.3333s-0.2666s) ~115° - 140°

= ½ * 35.7 * 0.1333 + ½ * 50.90 * 0.1333

+ ½ * 56.10 * 0.0333 + ½ * 44.95 * 0.0333

= 2.672 + 3.392 + 0.934 + 0.748 = **7.74** $\frac{m*L^2}{s^2}$ *s

As(t) = $\sum Ms(t)$ * α(rad) = 7.74 * 1.75 = **13.545** $\frac{m*L^2}{s^2}$ *s

During the whole duration of the fly cast the following work / energy is spent at the rod grip:

Af = Af(t) / t = 11.025 / 0.3333s = **33.07** $\frac{m*L^2}{s^2}$

As = As(t) / t = 13.545 / 0.3333s = **40.67** $\frac{m*L^2}{s^2}$

E3) Efficiency of the flexible and the absolutely stiff fly rod

The direct comparison of the efficiency of both rods, which are cast under the same constraints delivers the following:

η = E(rod tip) / A(rotation)

ηf(flexible fly rod) = Ef / Af = $\frac{78{,}56}{33{,}07}$ = **2.37**

ηs(absolutely stiff fly rod) = Es / As = $\frac{44{,}43}{40{,}67}$ = **1.09**

According to the initially stated constraints to both fly rods no mass is contributed. This constraint is the reason that the calculated efficiencies are higher than 1.0[30]. As the negligence of the mass has the same result for both fly rods, the comparison of the efficiencies ηf/ηs is still valid.

ηf / ηs = 2.37 / 1.09 = 2.17

13. Conclusion:* *The flexible fly rod transfers the energy introduced at the grip into tip more than twice as efficient as the absolutely stiff fly rod.

The work / energy that has to be spent by the fly caster also depends on the mass of the fly rod.

14. Conclusion:* *As less weight / mass the fly rod carries, as stronger its efficiency increases assuming the same rotational dynamics (keeping all other properties of the fly rod unchanged).

E4) Change of the efficiency during changing deflection

I would like to estimate shortly, how the efficiency changes with more or less deflection of the flexible fly rod. This analysis is only a rough estimation and does not intend to substitute a more exact calculation.

From my preceding investigations a good estimation can be gathered for the case, that the deflection of the flexible fly rod is smaller in comparison to the given one in my cast

[30] The efficiency η can not be bigger than 1.0, as the gained energy can never be bigger than the effort. When respecting the mass of both rods the torques in the grip would rise strongly in comparison to the calculated values, as that the efficiencies of both fly rods would be smaller than 1.0. The following estimation suggests: Assuming a weight of both fly rods of about 80 grams (which corresponds to the weight of the used fly rod SAGE 586 RPL+) and a weight of the elongated fly line of about 10 grams (which corresponds to the weight of the used fly line WF5F) leads to a factor 80g/10g = 8. If this mass ratio is approximately added to the force acting on the tip, also the moments as well as the effort work/energy rises by this factor. Then the efficiencies of both fly rods are below 1.0

sequence. In such case the values calculated for my cast sequence become more similar to the values calculated for the absolutely stiff fly rod, it means, the efficiency decreases.

The case of a higher deflection than the one given in my fly cast sequence is difficult to analyse due to lacking calculated values and due to the fact that my created deflection is quite big – in comparison to the fly cast sequences of other fly casters[31].

Surely the efficiency cannot be increased infinitely by rising deflection as it also depends on the possibility of the given fly rod to retract[32]. If the retraction capacity of the fly rod reached a certain limit, further increase of the effort (work) most probably would not show a visible bigger utility (final velocity).

At this point I would like to make a comparison to pole vault[33]: before 1900, when virtually stiff poles from the ash tree were used, the jumped height was limited to about 3 meters. With modern, flexible poles, about twice the height can be vaulted. The prerequisite for this, however, is that the deflection of the pole is well fitted to the weight (mass) of the pole jumper and his attempt velocity. Also this example shows, that flexible poles outmatch the stiff ones by far on the one hand, but also the deflection properties (retraction capacity) of the flexible pole plays an important role on the other hand.

On closer examination of my investigations I see an important reason for the efficiency of the flexible fly rod given by the fact that the work, the force, respectively, at the grip is spent by a short pulse "peak" in comparison to the absolutely stiff fly rod. This in turn, leads to a strong deflection of the flexible fly rod[34].

As long as the increasing deflection leads to an increase of this short pulse „peak“ of torque **and** an increase of the final velocity, as long the efficiency rises.

This estimation shows, that there is a correlation between the deflection of the fly rod and its efficiency. The value of the efficiency is mainly determined by the type and the magnitude of the deflection.

The following Figure XII shows – roughly estimated – the changes of the work at the rod grip during changing deflection:

[31] E.g. in comparison to the investigations of Løvoll & Jason Borger from 2007 – ‚The Rod & The Cast'. The deflection of my fly rod is much bigger than the deflection of the one investigated there.
[32] As faster a fly rod retracts into linear position as bigger the retraction potential is.
[33] As for other comparisons it also accounts for this one: Pole vault cannot and should not be equal to fly casting, because there are differences for sure. But there are parallel properties well picturing the impact of the efficiency during fly casting.
[34] To gain a short pulse of torque, the acceleration of the rod tip needs to be high for a short term. An essential prerequisite for this is a preferably small acceleration of the rod tip at the beginning of the fly cast. Therefore it is important, to initiate the fly cast slowly and to increase it towards the end. A more than linearily (nonlinear) rising acceleration of the rod tip is an advantage in such case.

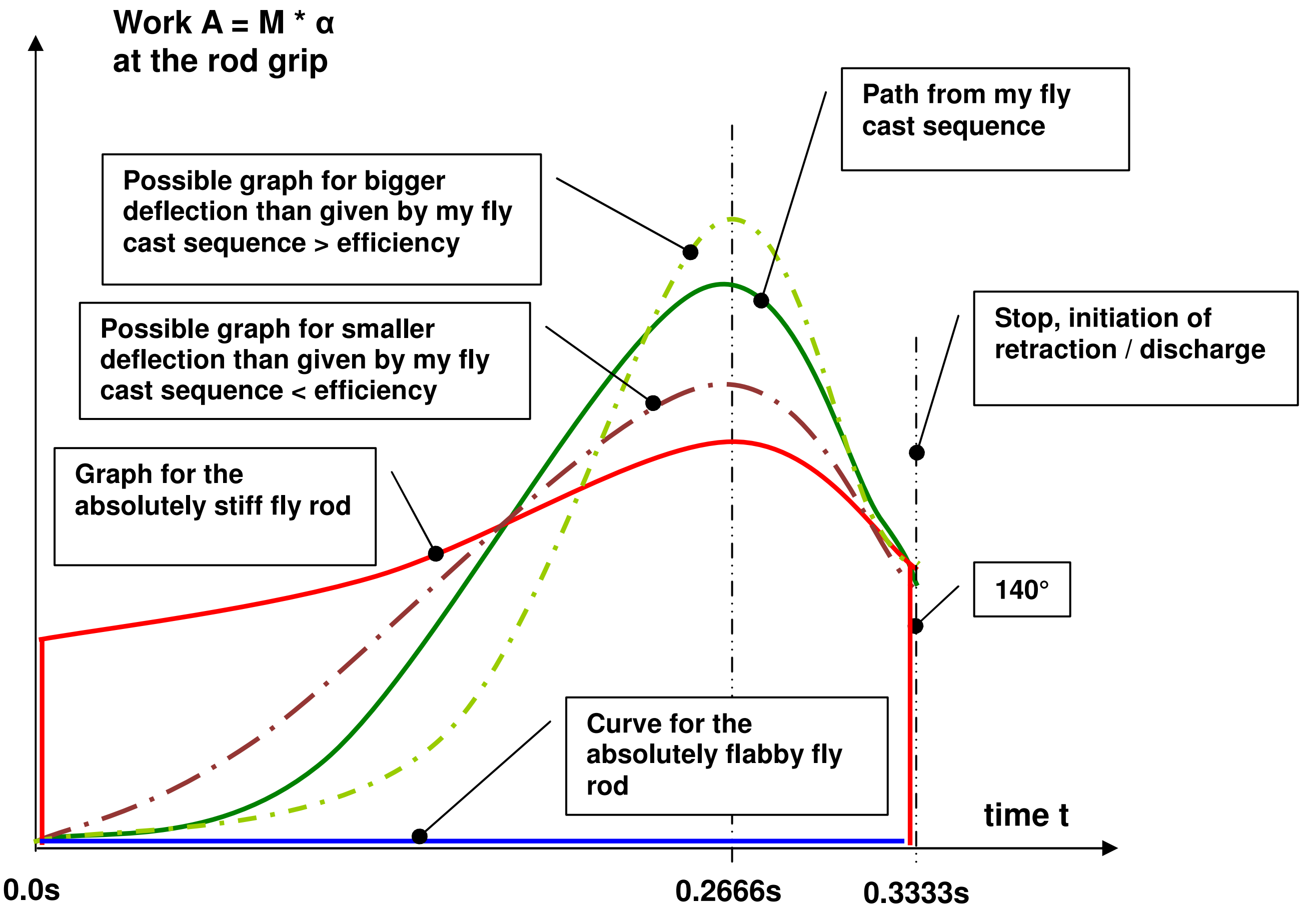


Figure XII

F) Investigation of further impact factors

The fly cast is influenced by other factors I want to elucidate shortly in the following. The further investigations do not claim completeness.

F1) Angular Momentum

The short term acceleration „peak" around the 90° position which had been stated in my previous investigations shows up, that the deflection of the fly rod leads to an additional effect which is based on the conservation of angular momentum[35].

[35] The investigations in section F), especially regarding angular momentum, were done in close collaboration with Dr. Schmitt (see acknowledgements). The conservation of angular momentum leads to the well known effect of the pirouette in figure skating and other sports.

Due to the rising deflection of the fly rod the turning point of the rotating mass[36] escapes the grip and moves upwards into the direction of the tip. This leads to an additional acceleration of the tip, i.e. the velocity rises strongly.

The angular momentum is based on the energy of rotation[37] and calculates according to the following formula:

$$D = m * r^2 * \omega;\ \text{mit}\ \omega = \frac{v(rodtip)}{r} \rightarrow D = m * r^2 * \frac{v}{r} = m * r * v$$

with D = Angular momentum; m = rotating mass; r = radius between center of the dynamics of the rotating mass and the tip of the rod; ω = angular speed around the center of the dynamics of the rotating mass.

The radius r has to be rectangular to the horizontal velocity (index h).

$$D = m * r,h^2 * \frac{v,h}{r,h} = m * r,h * v,h$$

The solution of the previous formula for v,h denotes to: $v,h = \frac{D}{m * r,h}$

The apex of the deflection of the fly rod is a suitable approximation for the center of the rotational dynamics of the mass and it is marked with the symbol ● in the following Figure XIII[38].

The angular momentum D is a physical invariant, that means during the acceleration of the fly rod the actually introduced angular momentum in the system is conserved when the rotating mass redistributes[39]. It remains in the system when the center of rotation is shifted. Therefore the angular momentum increases the velocity of the tip when the center of rotation is shifted upwards because the radius between center of rotation and tip shortens. For a simplified investigation we can assume, the angular momentum would be generally constant during the whole duration of the cast:

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[36] The ‘turning point’ of the rotating mass equals the ‘center’ of the rotating mass.

[37] Energy of movement, therefore kinetic energy, http://en.wikipedia.org/wiki/Angular_momentum

[38] A more detailed determination of the center of rotation is very complicated and only possible with higher mathematics. For the presentation of the mode of action of the conservation of angular momentum the described approximation is sufficient.

[39] As the rotation of the fly rod introduces continuously energy it is not a closed physical system on which the conservation of angular momentum is often demonstrated. But as the conservation of angular momentum is also valid during small sections of the whole cast, the redistribution of mass accelerates the tip when the “lever” is shortened during an acceleration process. This is mainly caused due to a shift of the rotating point during the cast. As long the energy which had been introduced into the fly rod shifts into the direction of the tip (as it is clearly visible from my cast sequence) the momentary angular momentum does not reduce and is conserved. The fact that this effect of redistributed angular momentum must exist is supported by the strongly accelerating tip in between the positions 40° to 90° and 90° to 140° when it accelerates from 5 % to 84 % of the final velocity (see 8th conclusion) corresponding to an acceleration to the 16-17 fold value. Therefore the acceleration of the tip is remarkably above the acceleration introduced into the grip. Dr. Schmitt (see acknowledgements) explains this in more detail in the annex 3 by further explanations regarding the angular momentum.

D(40°) = D(90°) = D(140°) = D(discharge) = constant

For the flexible fly rod it follows:

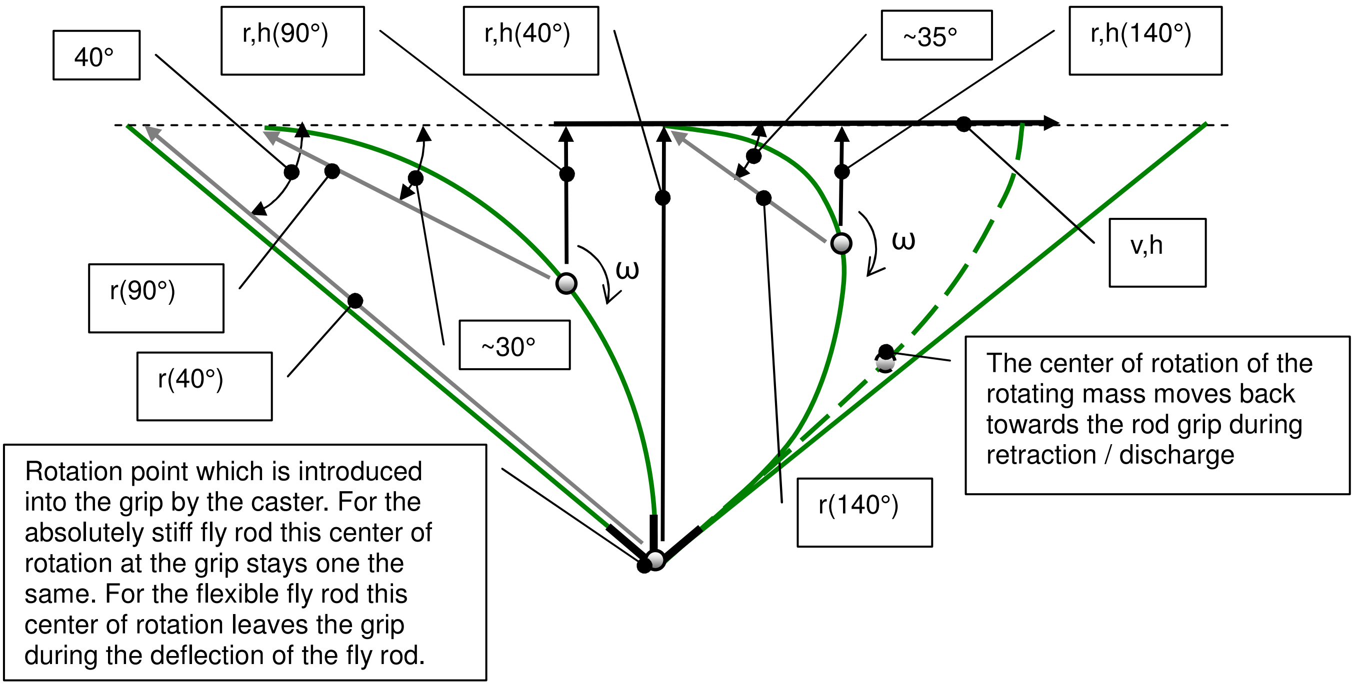


Figure XIII

The geometrical properties are determined from the previous Figure XIII[40]. The radius r can by approximated by:

r(40°) ~ L; r(90°) ~ ½ L; r(140°) ~ ⅓ L

The radius r,h rectangular to the velocity is calculated in the following table:

Position rotational angle / position of the rod grip	**40°**	**90°**	**140°**
Radius r of the angular momentum	**~ L**	**~ ½ L**	**~ ⅓ L**
Radius r,h of the angular momentum rectangular to the velocity v,h	**r,h ≈ r(40) * sin(40) = 0.643 L**	**r,h ≈ r(90) * sin(30) = 0.250 L**	**r,h ≈ r(140) * sin(35) = 0.191 L**

The moving mass m is located above the center of rotation at most. As the center of rotation moves towards the tip with increasing deflection in my fly cast sequence, the

[40] A more detailed determination of the geometrical relations is elaborate and would be beyond the scope of my investigations. This section shall elucidate the principal impact of the conservation of angular momentum. For that purpose the approximated geometrical relations are sufficient.

radius shortens and the moving mass reduces. For the moving mass m the following can be stated:

m(40°) > m(90°) > m(140°)

For the sake of simplicity this positive effect of the reduction of the moving mass m is neglected and it is assumed that the mass is constant. The angular momentum acts onto the horizontal velocity v,h of the rod tip as follows:

v,h(>40°) = $\frac{D}{m*0.643L}$; v,h(90°) = $\frac{D}{m*0.25L}$;

v,h(140°) = $\frac{D}{m*0.191L}$; v(discharge) = $\frac{D}{m*0.643L}$

v,h(90°)/v,h(>40°) = $\frac{0.643}{0.25}$ = **2.57**; v,h(140°)/v,h(90°) = $\frac{0.25}{0.191}$ = **1.31**

v(discharge)/v,h(140°) = $\frac{0.191}{0.643}$ = **0.30**

***15. Conclusion:* The impact of the angular momentum alone leads to an increase of the horizontal velocity of the rod tip by a factor of 2.57 between the 40° and the 90° position and another additional factor 1.31 between the 90° and the 140° position. Especially when the deflection increases, the horizontal velocity increases significantly. The decrease of this velocity (according to the conservation of angular momentum) by a factor of 0.30 during the retraction / discharge is compensated by the initiating tension release of the fly rod (spring effect)[41].**

For the absolutely stiff fly rod this effect of the angular momentum does not exist because it does not deflect. Its center of rotation of the rotating mass stays in the rod grip. It falls together with the rod grip.

[41] Due to the previously described, simplified assumptions the factors do not claim to represent exactly the increase of the velocity of the rod tip. The interplay between the introduced rotational energy, the shift of the center of rotation and the potential tension energy leads to the fact that the tip of the flexible fly rod moves slowlier than the tip of the absolutely stiff fly rod but starts to catch up due to the shift of the center of rotation in spite of the further rise of the potential tension energy. Already in the 90° position the tip of the flexible fly rod reaches the speed of the tip of the absolutely stiff fly rod. Beyond this position the speed of the tip of the flexible fly rod passes the speed of the absolutely stiff fly rod and finally reaches a value 33 % higher than the speed of the absolutely stiff fly rod (see 7th conclusion). For the flexible fly rod most of the energy is transferred at the end of the fly cast.
The deflection causes a shortening between the rotating mass (which especially is represented by the tip segment and the fly line) and the rotation point at the grip. Because the angular momentum, which is already present in the system redistributes, this shortening must lead to an additional rotation speed – similar to a figure skater who increases the rotation speed of the pirouette by attracting his arms. As the fly rod deflects near the grip too further effects of the redistribution of the mass and therefore inertia of the fly rod are expected. The deflection brings mass elements near the grip closer to the rotation point and a further redistribution of angular momentum occurs. This impact of the angular momentum caused by the decreasing inertia could be clarified by an additional shortening between the rotating mass and its rotation point respectively by a shift of the rotation center towards the tip of the fly rod.

For a very stiff fly rod which deflects only a little, the center of rotation of the rotating mass moves a bit from the grip towards the rod tip, however it does not reach the large distances from the grip like in my fly cast sequence. Therefore in such case the decrease of the radius r and r,h, respectively, is smaller as also the increase of the velocity due to the conservation of angular momentum[42].

The effect of the conservation of angular momentum is in agreement with my estimation in section E4), that the efficiency reduces when the deflection decreases. Moreover the investigation shows that the increasing deflection until the initiation of the stop, not only increases the potential tension energy of the fly rod but also strengthens the effect resulting from the conservation of angular momentum[43].

F2) The parallel translation

The distance of the position of the rod grip between the stop positions (initial and final position) indicates, how strongly the parallel translation is applied by the fly caster. As bigger as this distance is, as more the parallel translation is inserted into the fly cast. It prolongates the pathway, the rod tip propagates – in comparison to its pathway during exclusive rotational movement.

F2.1) Parallel translation in the fly cast sequence

From my fly cast sequence it becomes visible, that I infer a comparatively long parallel translation (see pictures in section C2). It measures about one meter[44]. The pathway length of the rod tip prolongates by this about one meter. The rod tip moves along a distance of 1.54 * L (see section B3) due to the rotational movement.

At a length of 2.65m, as it is the case for the used SAGE 586 RPL+, this rotational movement measures to:

WRs(f) = 1.54 * 2.65m = **4.08m**

Therefore the proportion of the parallel translation in my casting sequence is 1.0m / 4.08m ≈ ¼ of the whole pathway of the rod tip. It attracts attention that this proportion decreases for longer fly rods and increases for shorter fly rods, respectively.

[42] The fly rod concentrates / condenses or "pumps" the introduced kinetic energy into its tip via the effect of the conservation of angular momentum and the described redistribution of the rotating mass which reduces into the direction of the tip. The absolutely stiff fly rod can not do this.

[43] The fact that the deflection increases until the initiation of the stop, the retraction / discharge respectively, was pointed out by the 9th conclusion. As long as the deflection of the fly rod increases in the same way as stated in my fly cast sequence, also the radius shortens. The shortening of the radius increases the effect of the conservation of angular momentum. The investigation of the angular momentum shows, that an increasing deflection of the fly rod can be advantageous for the fly caster.

[44] The parallel translation is estimated from the distance of the rod grip between the first and the last picture of my fly casting sequence. The reference is given by my body height of 1.77 m.

16. Conclusion: ***As shorter the fly rod is, as more the parallel translation contributes to the total, horizontal path of the fly rod tip. As longer the fly rod is, as less the parallel translation contributes to the total, horizontal path of the fly rod tip[45].***

From my casting sequence it becomes evident furthermore, that the parallel translation is conducted mainly at the beginning of the fly cast. Therefore a possibly existing part of a "slack fly line" at the beginning of the fly cast is eliminated, the fly line is connected linearly to the rod tip ("force coupling") and the fly rod is preloaded for the subsequent rotational movement (see pictures 1 to 6 in section C2). The deflection of the fly rod is activated directly at the beginning of the fly cast and can be increased smoothly[46].

This additional path from the parallel translation delivers advantages for the fly caster. The fly caster of the absolutely stiff fly rod could move along a linear part of the path of the rod tip employing the parallel translation. The parallel translation also secures, that the center of rotation of the fly cast is located at a lower point (see following Figure XIV). As lower the center of rotation as more the linear part of the path of the rod tip can be increased while keeping small the rotational angle. This is a big advantage for a narrow loop.

As the translational shift operates similar for both rods it is inferred that it´s influence annihilates when doing a comparison.

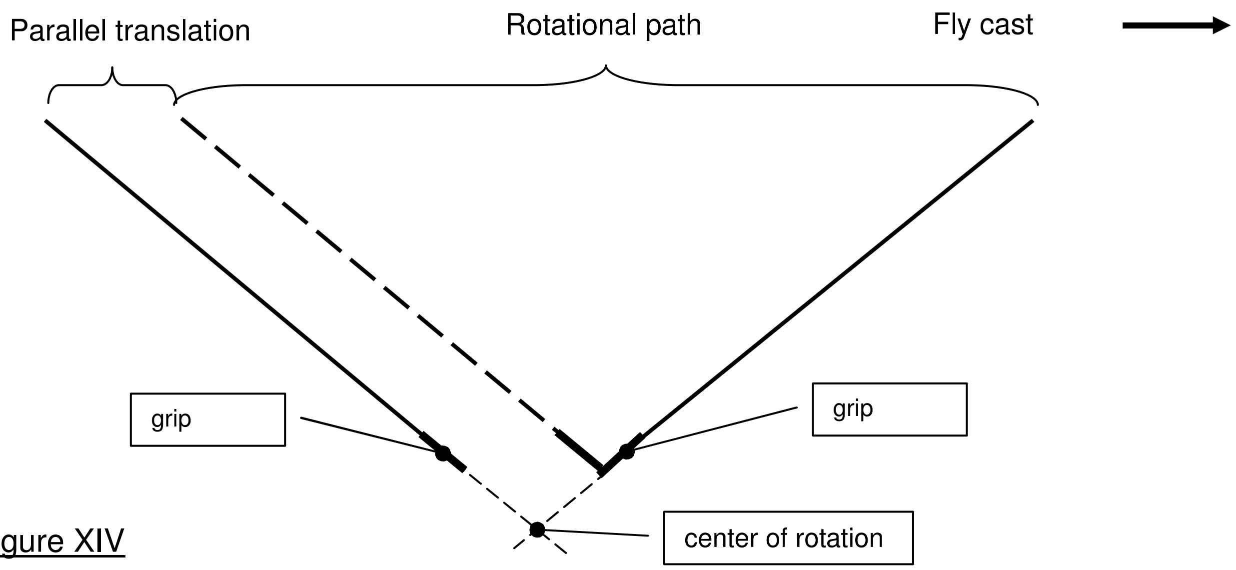


Figure XIV

[45] Hereby it also becomes clear, that a pronounced parallel translation is a big advantage for shorter fly rods. "Short sticks" (fly rods around 2 meters) therefore are thrown „from the shoulder" with inactive wrist - see e.g. Hans Gebetsroither. Therefore the rotational center shifts downwards and the proportion of the parallel translation increases – by this the path of the rod tip can increase significantly especially for those short fly rods.

[46] For that reason also the efficiency of other special fly castes like e.g. the „roll cast" ("half" fly cast) or Switch Cast (also named D- Cast) can be increased significantly by a pronounced parallel translation.

F2.2) Pure parallel translation for the absolutely stiff fly rod

According to the initially stated prerequisites I have assumed a rotational movement for the absolutely stiff fly rod. From this follows a significant disadvantage according to the decomposition of the forces I described in chapter B4)[47]. In the following I want to estimate shortly, if the absolutely stiff fly rod could have become more efficient, if this disadvantage would be erased.

The described disadvantage would be omitted, if the absolutely stiff fly rod could be cast with a pure parallel translation. The pure parallel translation is shown in Figure XV in comparison to the investigated rotational movement.

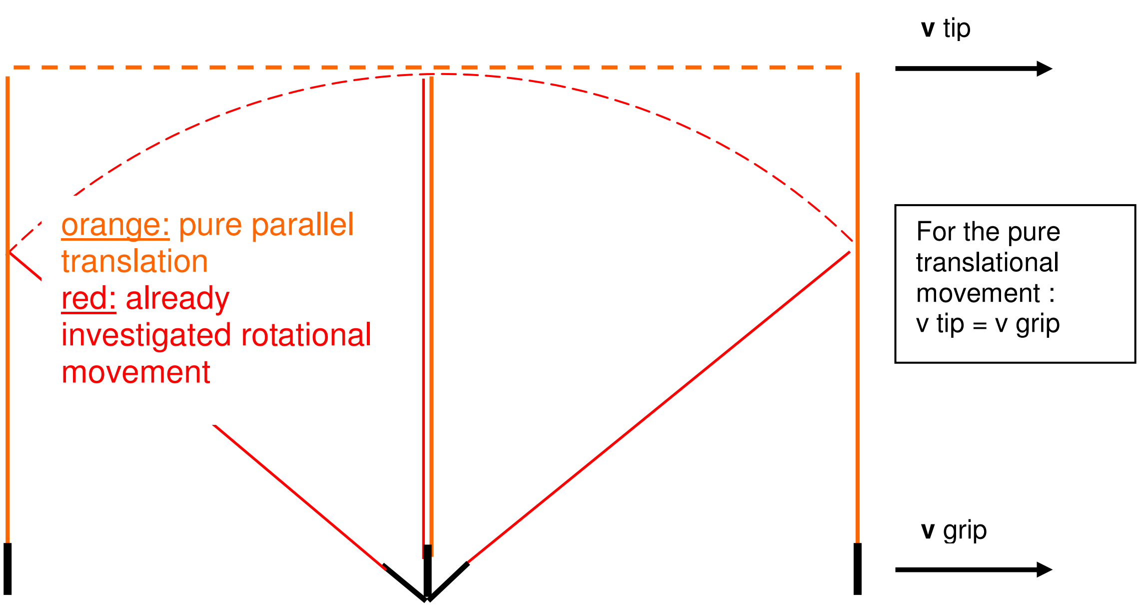


Figure XV

In such a case the velocities of the tip and the grip of the absolutely stiff fly rod would be equal. According to the relation E = ½ * m * v², the equal velocities create the same energy. Therefore the energy which is inferred into the grip by the fly caster must be the same like the energy which is transferred to the rod tip:

η(translation) = E(rod tip) / E(grip) = 1.0

With 1.09 the efficiency calculated in section E3) for the rotational movement of the absolutely stiff fly rod is virtually the same. At parallel translation only the efficiency of the absolutely stiff fly rod would not be higher than at a rotational movement, I proposed in my investigations and compared with the results of the flexible fly rod.

[47] On the other hand the absolutely stiff fly rod gains an advantage from the rotational movement that the lever arm shortens and therefore the effort for the fly caster reduces (see sections D3 and E2).

F3) Other dynamic courses of the absolutely stiff fly rod

As described in section F2.2) the absolutely stiff fly rod is not able to transfer the introduced work or energy, respectively into the tip more efficiently than the flexible fly rod in my cast sequence neither when moved by parallel translation only nor when moved rotational only. Here I want to give an estimation if any other dynamic course, another pathway or time dependency of the velocity could change this.

The tip of the absolutely stiff fly rod could be moved along a straight path instead a circle segment if the caster introduces a compensating movement (a vertical swing). This dynamic course is presented in the next Figure XVI. The comparative calculation for this case (see annex 1) showed that the final velocity v(end) of the rod tip as well as the acceleration rise significantly. The rising acceleration leads to rising forces at the rod grip – and therefore to a rising effort of the fly caster. Due to the vertical swing the caster has an additional effort so that finally the efficiency approaches a value of 1.0[48]

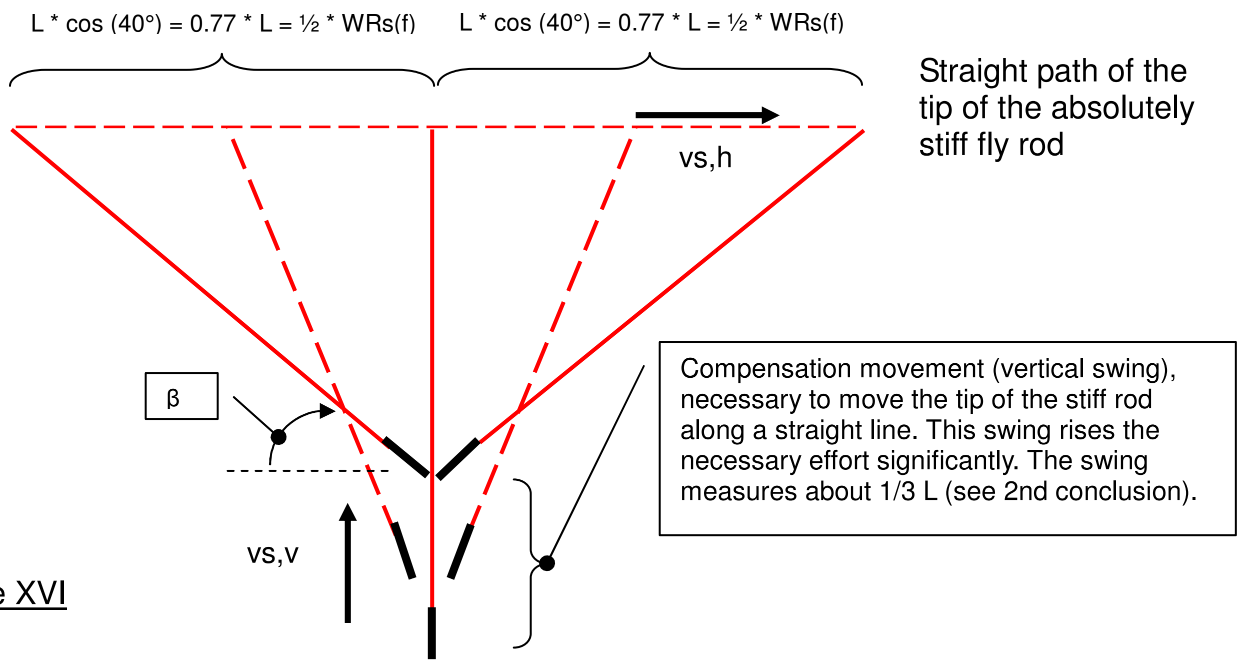


Figure XVI

[48] The comparative calculation in annex 1 was done under the same constraints and with the same formulas as done in my previous calculations. As a result the absolutely stiff fly rod can not reach a higher efficiency than calculated before. Indeed the final velocity of the rod tip reaches a value of 13 L/s but even if only the effort for the rotation is respected the efficiency of the absolutely stiff fly rod is lower than the efficiency of the flexible fly rod. If the effort for the vertical swing is also taken into account the value for the efficiency of the stiff fly rod approaches 1.0 and the ratio of the efficiencies of both fly rods ηf/ηs shifts towards the flexible fly rod and reaches the value calculated in section E3. As the efficiency for the massless absolutely stiff fly rod is not exceeding 1.0, also the absolutely stiff fly rod including the mass can not reach the efficiency of the flexible fly rod – independent from the way the dynamics is performed. For the flexible fly rod the efficiency depends on the deflection and is therefore varying.

The dynamics investigated here shows that the absolutely stiff fly rod works similar to a connecting rod[49]. The connecting rod transfers the energy which is introduced into one end (here: grip) "one by one" 1:1 into the other end (here: tip) and does not have effects which are positive for the transfer of energy into the rod tip velocity only. This similarity explains why the absolutely stiff fly rod is unable to rise its efficiency.

***17. Conclusion:* Neither by pure rotational nor by pure parallel translational movement the efficiency of the massless absolutely stiff fly rod can be increased to values above 1.0. The absolutely stiff fly rod can not deliver an improved transfer of energy also for other pathways, rotational angles or dynamical paths of the velocity. For that reason the absolutely stiff fly rod does not get any disadvantage due to the rotational movement used in my investigations[50].**

To be able to move the tip of the absolutely stiff fly rod the caster has to perform a large translational movement and a small rotation. If he intends to span the same angle of 100° like in my investigations, he has to compensate with a swing of about 1/3 of the rod length in the vertical 90° position (see 2nd conclusion and Figure XVI). For the used SAGE 586 RPL+ with a length of 2.65 meters this swing measures 88 cm in total.

***18. Conclusion:* It is the rotational movement especially which rises the velocity of the tip at most. The caster of the absolutely stiff fly rod is missing this advantage especially for large rod lengths and bigger rotation angles, because he has to leave the straight path of the tip or he must compensate with a vertical swing of the grip what is getting more difficult and finally impossible with rising rod length[51].**

[49] Also called plunger rod. Generally such rods transfer energy into a crank shaft. They also can transfer energy along a straight path for example – as it is described by the straight line of the tip of the absolutely stiff fly rod for the dynamics described above.

[50] Even if higher tangential velocities are investigated (see section 3.2) the efficiency can not rise, as due to the corresponding geometry the effort is rising further (full rod length as a lever arm and the higher tangential part of the force acting onto the tip rise the effort at the grip, see section B4.2). As any possible dynamics can be decomposed into translation and rotation for all stiff systems the efficiency is invariant to combinations of dynamics. Consequently the absolutely stiff fly rod can not rise its efficiency if the cast is performed with other, for example smaller, rotation angles than the angles assumed in my investigations. For the efficiency of the absolutely stiff fly rod the following experiment in minds: A compact mass object – for example a marble / sphere – shall be accelerated. To get this object into movement exactly the amount of energy is needed as this object later exhibits in form of kinetic energy (rotation and translation). The efficiency (ratio between the energy of its movement and the introduced energy) will be unchanged. If this object is an elongated structure as a 'stick' or an absolutely stiff fly rod, this can not affect the ratio of energies.

[51] It becomes evident that the absolutely stiff fly rod can not get rid of a big disadvantage : The vertical part of the movement ! If a pure rotational dynamics is performed in the rod grip the tip is deviating from the straight path ("windscreen wiper"). If the rod tip is moved along a straight path on the other hand, a compensating swing in the rod grip becomes necessary (see Figure XVI). Both vertical dynamics reduce the efficiency of the stiff fly rod ! Only a pure translational movement would be suitable to move the tip of the absolutely stiff fly rod without vertical part of the movement along a straight path (see section F2.2). However in such a case the whole rod has to be moved with the same velocity of the tip, a big disadvantage. Consequently the translational dynamics is not able to rise the efficiency. The flexible fly rod on the other hand is able to avoid nearly completely the disadvantageous vertical movement, resulting in a significant reduction of the casters effort.

F4) Damping

The damping properties as well as the properties of the post-pulse oscillation are very different for the flexible and the absolutely stiff fly rod.

The flexible fly rod rises its potential tension force due to deflection until the retraction / discharge initiates. This tension force is released continuously during retraction / discharge (see section D1, Figure X). Similar to that the effort of the caster rises or reduces, respectively. At the end of the retraction / discharge the tip of the flexible fly rod passes its equilibrium point due to its velocity and mass (inertia) and builds up a small counterdeflection into the other direction[52].

The continuous buildup and reduction of tension force is significantly damping the energy system of the flexible fly rod. Force and energy pulses can be damped by the fly rod. Therefore the force effort of the fly caster is harmonic – means it approaches to a harmonic sequence.

Due to the missing deflection the absolutely stiff fly rod can not establish potential tension (see section D2). Therefore it also has no damping properties. Its mass has to be accelerated and stopped by a sudden force effort. The force effort of the fly caster is not harmonic but disharmonic.

The damping rises the sense of well-being, it is comfortable. To illustrate this behavior I would like to make a comparison to a mattress: according to personal preferences there exist soft and hard mattresses[53]. They all have common to cushion the movement impulses by accommodating pressure points of the body. The body can lie comfortable and without trouble.

Similar to the usage of a mattress the fly caster of the flexible fly rod profits from the comfort of damping and he is able to cast without pain for a long duration. The caster of the absolutely stiff fly rod will not be without trouble. He dispenses with comfort of a mattress, his body lies on a hard plank for comparison. He will neither lie with comfort nor he will be able to change his position without getting black and blue marks[54].

[52] This first counterdeflection can not be avoided and leads to an initially somewhat larger loop as compared to the absolutely stiff fly rod. The first counterdeflection has to be judged as a disadvantage – due to a small loss of energy, resulting from the elastic properties of the fly rod. All further counterdeflections should be suppressed by the fly caster. This will result in an oscillation free loop image, which is important for an optimized transfer of energy. Due to many significant advantages of the flexible fly rod this disadvantage should not be very severe.
As described before the flexible behavior causes a small loss of energy. It is estimated that this loss applies about 1/3 of the potential energy. As the portion of the potential energy lies around 1/4 of the whole energy (see section D1), the total loss makes up only ~8% (1/4 * 1/3 = 1/12 = 0,083 = 8,3%) and could be neglected.

[53] As compared to fly rods which have harder or softer action.

[54] Casters of stiff, nearly not deflectable rods reported that the casting arm started to hurt after a short time. The caster of the stiff fly rod is therefore facing health problems.

G) Conclusion and final investigations

The results of my investigations involve the analysis of my cast sequence and the amount of fly line casted (see section A). It remains open if an analysis of other fly casts would deliver similar results[55].

<u>Final Conclusion:</u> My investigations show, that the deflection of the fly rod leads to advantages during the whole duration of the fly cast. These advantages result mainly from the interplay between the conservation of angular momentum and the spring tension energy. The parallel translation can help to use these effects better. According to the missing deflection the absolutely stiff fly rod does not show such advantages. The flexible fly rod transfers the energy which is inferred into the grip much more efficient into the tip than the absolutely stiff fly rod can do.

G1) Uncertainties / approximations

Standing alone the angles calculated due to the assumed constraints (see section A) have to be judged with care. As the dynamic and geometrical properties derived from my casting sequence are simplified and both rods do not carry mass an accurate analysis of both fly rods will result in somewhat differing results. Especially due to the assumption of massless fly rods forces and energies which especially result from the movement of the mass are not respected. This assumption results from my intention to keep the investigations and calculations as simple as possible to reach a larger audience. The preface shows that also Dr. Schmitt agrees with this concept.

Especially because the assumed massless fly rods influence my conclusions at most I have made an estimation of its influence on the cast in annex 2. The results show that also an inclusion of the mass of both fly rods does not significantly influence my conclusions.

The approximations resulting from the given constraints can not question my conclusions in total. All further estimations show that this lead neither to preference of the absolutely stiff nor the flexible fly rod.

The approximations resulting from the fact that only three positions of the fly rod were investigated and the values in between were interpolated are more difficult to judge. This might lead to inappropriate results especially for the flexible fly rod because its behavior can not be described by a linear course at most. Surely my investigations would have been more exact if further positions would have been evaluated but this would have been beyond the scope of this study and beyond my technical options. At this point I have to suppose that the investigated positions are characteristic to image the reality. This is supported by the fact that my results are widely consistent.

[55] Many indications point out that small deviations from tension dynamics as compared to my casting sequence reduce the efficiency of the flexible fly rod. If the caster sets pressure points or if he is overpowering the cast, the efficiency of the flexible fly rod reduces inevitably. The efficiency depends "sensibly" on the casting sequence and demands therefore an accurate sequence from the caster. Each path introduced by the caster not resulting in a continuously rise of the deflection does not lead to a rising efficiency.

Even if the calculated values would shift disadvantageously for the flexible fly rod at more exact evaluation the shown advantages in comparison to the absolutely stiff fly rod would remain.

G2) Summary

From my investigations mainly the following advantages result, a flexible fly rod has in comparison to an absolutely stiff fly rod:

- For the same effort the final horizontal speed of the tip of the flexible fly rod is significantly larger than for the absolutely stiff fly rod.
- The fly caster can initially slowly increase the force, the work, respectively for the flexible fly rod and release it at the end of the fly cast (harmonic behavior). The absolutely stiff fly rod needs a prompt effort of force, work, respectively, at the beginning of the cast, which promptly decaying at the end of the fly cast (disharmonic behavior).
- Due to the damping properties the flexible fly rod enables a casting which is free of trouble / pain and comfortable for the caster. This can not be provided by the absolutely stiff fly rod.
- Different to the flexible fly rod the caster of the absolutely stiff fly rod must keep the working angle small or conduct a vertical compensation (swing) in the rod hand, to guide the tip on an approximately straight path. Both is limiting the casting possibilities of the absolutely stiff fly rod.
- The deflection of the flexible fly rod shows two positive effects via the angular momentum (concentration effect of kinetic energy) and the spring tension, which are not found for the absolutely stiff fly rod.
- For the flexible fly rod the most intense force, work, respectively, needs to be spent only for a very short time period („peak“) during the cast.
- ***The efficiency of the flexible fly rod in the transfer of energy introduced from the grip to the tip is much better than the efficiency of the absolutely stiff fly rod – even if all approximations are respected.***

If only the flexible fly rod is analyzed solely, many advantages offered the fly caster by the deflection are not clearly visible. Just the comparison between the flexible and the absolutely stiff fly rod show the advantages of the flexible fly rod. Without doubts also the absolutely stiff fly rod can cast a fly line. But the fly caster has to spend more force, work, respectively for the same end velocity of the rod tip and he must accept further disadvantages and even face healthy risks (see for example section F4).

Especially due to the properties of the deflection the flexible fly rod can increase the efficiency in comparison to the absolutely stiff fly rod. It is important that the weight of the fly line and the desired final speed is adapted to the retraction potential of the fly rod.

***As better the caster uses the deflection of the fly rod in the range of its capacity, as more efficient the fly rod transfers the work introduced by the caster into its tip*[56].

Acknowledgements and final remarks

My gratitude belongs to

a) Especially the physicist Dr. Franz-Josef Schmitt, who had advised and supported me regarding all physical context,
b) The certified Flycasting instructor and mathematician Dr. sc. math. Jean-Paul Kauthen, who has given me precious advises for the whole duration of elaboration of the first version 1.0.

Without your elaborated help my investigations could not have been presented with such precise formulations.

You, Franz- Josef, supported me a lot by analyzing all comments which were observed after the first draft of this work has been published in february 2014. You explained the impact of the angular momentum to me, which wouldn't have been represented in that form without your deep knowledge. My special thank belongs to you[57].

Finally I thank my meanwhile three years old twin daughters Greta & Theresa. Because you sleep the whole night since your 4th month I was able to find the necessary time and concentration in the evening for my investigations. I hug you !

Altogether with interrupted time slots I worked more than one year on these investigations until I finalized the first draft in early February 2014[58]. During this time span I had some „Eureka"-effects that taught me the physical requirements for an efficient, force minimizing fly cast. These insights will be incorporated into my work as a certified Fly Casting Instructor !

[56] My investigations underline and prove the statements of the teaching film „perfect fly casting" (german:" Perfektes Fliegenwerfen"), which was produced by H.R. Hebeisen together with me in 2009. By the way Dr. Schmitt (see acknowledgements) made the remark, that evolution always leads to the survival of the best technique if it is used often. My investigation present the essential reasons why we fly fishers work with the flexible instead of the absolutely stiff fly rod!

[57] As the translator of my german speaking investigations Dr. Schmitt renounced to translate this section. In order to appreciate his support I decided to translate this note of thanks by my own.

[58] As compared to the first draft from february 2014 in my present revision 2.0 mainly section F was added. The present revision 2.0 was released in november 2014. I also would like to mention that the users "Laverda" and "Merlin" from the forums "Leidenschaft-Meerforelle.de" and sexyloops.com have contributed with their hints and suggestions so that I thought about the conclusions of my investigations more deeply. The suggestions are respected in the present revision 2.0 and finally convinced me even more of the advantages of the flexible fly rod.

I am aware that fly casting is a very complex physical problem when treated more exactly. It was my main aim to have a short look on the physical values and effects which act on the fly rod during the cast. I would be pleased if my investigations could contribute to better understand the physical context of fly casting.

Potsdam-Rehbrücke in february / november 2014

Tobias Hinzmann

Annex 1: comparative calculation for section F3 (vertical adjustment)

Velocity of the tip of the absolutely stiff fly rod (see section C3.2)

Position rotation angle / position of the grip	**90°**	**140°**
Passed distance of the rod tip (absolutely stiff)	***0.77 * L***	***1.54 * L***
Differences	**40° bis 90°**	**90° bis 140°**
Horizontal velocity vs,h of the rod tip	**vs,h = (0.77-0.00)L /0.2666s = *2.888L/s***	**vs,h = (1.54-0.77)L /0.0666s=*11.561L/s***

The geometrical relations can be taken from Figure XVI.

$$\frac{0.5*((0.2666-0)+(0.3333-0.2666))}{(11.56-2.888)} = \frac{0.5*0.2666}{vs(90°)-2.888}$$

$$\rightarrow \text{vs,h}(90°) = \frac{0.5*0.2666*8.672}{0.1666} + 2.888 = 6.938 + 2.888 = \mathbf{\mathit{9.826}}\ \text{L/s}$$

$$\frac{0.5*((0.3333-0.2666)+(0.40-0.3333))}{(11.56-9.826)} = \frac{0.5*(0.3333-0.2666)}{vs,h(140°)-11.56}$$

$$\rightarrow \text{vs,h}(140°) = \frac{0.5*0.0666*1.734}{0.0333} + 11.56 = 1.734 + 11.56 = \mathbf{\mathit{13.294}}\ \text{L/s} = \text{vs,h(end)}$$

Acceleration of the tip of the absolutely stiff fly rod (see section C4)

$$\text{as1} = a(40°\text{-}90°) = \frac{2.888-0}{0.5*(0.2666-0)}\ \frac{L}{s^2} = \frac{2.888}{0.1333} = \mathbf{\mathit{21.66}}\ \frac{L}{s^2}$$

$$\text{as2} = a(90°) = \frac{11.56-2.888}{0.5*((0.2666+0.3333)-0.2666)}\ \frac{L}{s^2} = \frac{8.67}{0.1666} = \mathbf{\mathit{52.05}}\ \frac{L}{s^2}$$

$$\text{as3} = a(90°\text{-}140°) = \frac{13.294-9.826}{0.3333-0.2666}\ \frac{L}{s^2} = \frac{3.468}{0.0666} = \mathbf{\mathit{52.07}}\ \frac{L}{s^2}$$

Forces acting on the tip of the absolutely stiff fly rod (see section D2)

The mass m of the fly line is accelerated simultaneously by rotation and by the vertical compensation (swing). The part which is delivered for the acceleration of the mass m by the different dynamics is varying with the rotation angle β. Both parts can be calculated from geometrical relations (see Fig. VI in section B4.2):

m(rotation) = m * sin(β); m(vertical) = m * cos(β)

From this the forces can be calculated resulting from the acceleration of the mass m from rotation (Fs) and the vertical swing (Fv).

Fs0 ≈ m * sin(40) * a1 = ***13.922*** $\frac{m*L}{s^2}$ = Fs(>40°) (see 10th conclusion)

Fs1 = m * sin(65) * a1 = ***19.630*** $\frac{m*L}{s^2}$ = Fs(40°-90°)

Fs2 = m * sin(90) * a2 = ***52.05*** $\frac{m*L}{s^2}$ = Fs(90°)

Fs3 = m * sin(115) * a3 = ***47.191*** $\frac{m*L}{s^2}$ = Fs(90°-140°);

Fs4 = m * sin(140) * a3 = ***33.469*** $\frac{m*L}{s^2}$ = Fs(140°)

Forces (torques) at the grip of the fly rod (see section D3)

Ms0(>40°) ≈ 13.922 * 0.64 L = ***8.91*** $\frac{m*L^2}{s^2}$

Ms1(40°-90°) = 19.630 * ½ (0.64 + 1.0) L = ***16.097*** $\frac{m*L^2}{s^2}$

Ms2(90°) = 52.05 * 1,0 L = ***52.05*** $\frac{m*L^2}{s^2}$

Ms3(90°-140°) = 47.191 * ½ (1.0 + 0.64) L = ***38.696*** $\frac{m*L^2}{s^2}$

Ms4(140°) ≈ 33.469 * 0.64 L = ***21.417*** $\frac{m*L^2}{s^2}$

Work / energy, respectively from rotation (see section E2)

$\sum Ms(t)$ = ½ (M0 + M1) * ½ * 0.2666s + ½ (M1 + M2) * ½ * 0.2666s

+ ½ (M2 + M3) * ½ * (0.3333s-0.2666s) + ½ (M3 + M4) * ½ * (0.3333s-0.2666s)

= ½ * 25.007 * 0.1333s + ½ * 68.147 * 0.1333s + ½ * 90.746 * 0.0333s

+ ½ * 60.113 * 0.0333s = 1.671 + 4.542 + 1.511 + 1.001 = ***8.725*** $\frac{m*L^2}{s^2}$ *s

As(t) = $\sum Ms(t)$ * α(rad) = 8.725 * 1.75 = ***15.268*** $\frac{m*L^2}{s^2}$ *s

As = As(t) / t = 15.268 $\frac{m*L^2}{s^2}$ *s / 0.3333s = ***45.808*** $\frac{m*L^2}{s^2}$ (As only from rotation !)

Work / energy, respectively of the vertical swing (see Figure XVI)

In the Following only the work / energy from the upwards directed swing is analyzed which has to be spend between the 90° and 140° position. The work / energy of the downwards directed vertical movement between the 90° and 140° position is neglected as it is of minor relevance (the downwards directed movement is performed during significantly smaller acceleration during an additional longer time span). Due to this neglecting a part of the real work / energy that has to be spent by the caster is not respected. Therefore this calculation is on the safe side. The vertical velocity of the grip (swing velocity) vs,v and the horizontal velocity of the tip vs,h have the following relation: vs,v = vs,h * cos(β)

$$vs,v(90°) = vs,h(90°) * \cos(90) = 0.00 \ \frac{L}{s};$$

$$vs,v(140°) = vs,h(140°) * \cos(140) = 0.766 * 13.294 = 10.183 \ \frac{L}{s}$$

Vertical acceleration a,v of the grip: a,v = Δvs,v / Δt

$$a,v(140°) = (10.183 - 0.00) \frac{L}{s} / (0.3333\text{-}0.2666)s = 152.89 \ \frac{L}{s^2}$$

The part of the mass m which is accelerated by the angle connection is calculated according to Figure VI, section B4.2 to m(vertical) = m * cos(β).

As the horizontal acceleration of the rod tip does not change significantly between the 90° and the 140° position (and therefore can be assumed to be constant – see presentation of the acceleration in Figure IX) also the vertical acceleration of the grip can not change significantly. From a,v ~ constant it follows:

$$a,v(>90°) \sim a,v(90°\text{-}140°) \sim a,v(140°) \sim 152.89 \ \frac{L}{s^2}$$

$$F,v\ (90°) = m * \cos(90°) * 152.89 \ \frac{L}{s^2} = 0.00 \ \frac{m*L}{s^2}$$

$$F,v\ (90°\text{-}140°) = m * \cos(115°) * 152.89 \ \frac{L}{s^2} = 64.614 \ \frac{m*L}{s^2}$$

$$F,v\ (140°) = m * \cos(140°) * 152.89 \ \frac{L}{s^2} = 117.120 \ \frac{m*L}{s^2}$$

The vertical work (energy) As,v that has to be spent additionally by the caster of the absolutely stiff fly rod, calculates according to the work relation As,v = „force * distance“, where the distance measures as the vertical dynamics (swing):

$$As,v = F,v(90°\text{-}140°) * ½ * 0.36\ L + F,v\ (140°) * ½ * 0.36\ L$$

$$= 64.614 \ \frac{m*L}{s^2} * 0.18L + 117.89 \ \frac{m*L}{s^2} * 0.18L = 32.850 \ \frac{m*L^2}{s^2}$$

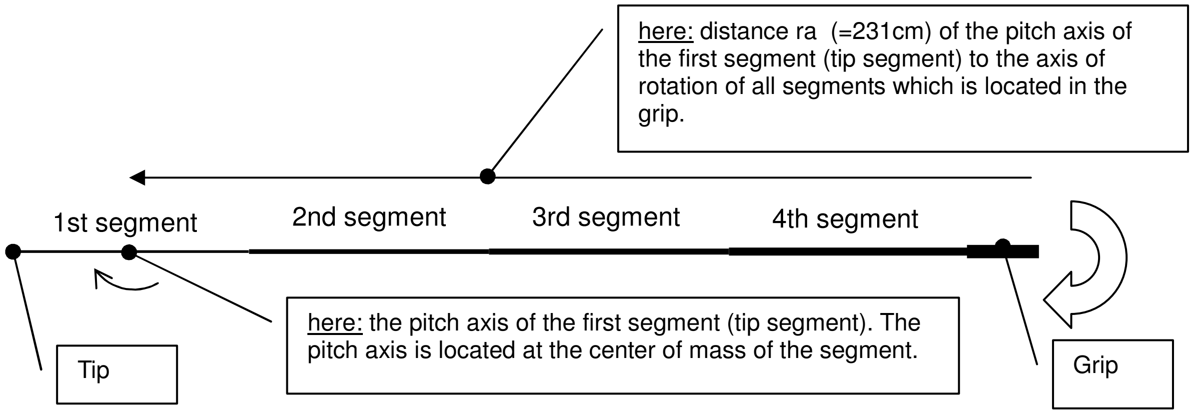
$$As,total = As + As,v = 45.808 + 32.850 = 78.658 \ \frac{m*L^2}{s^2}$$

Efficiency of the absolutely stiff fly rod, if its tip is guided along a straight path:

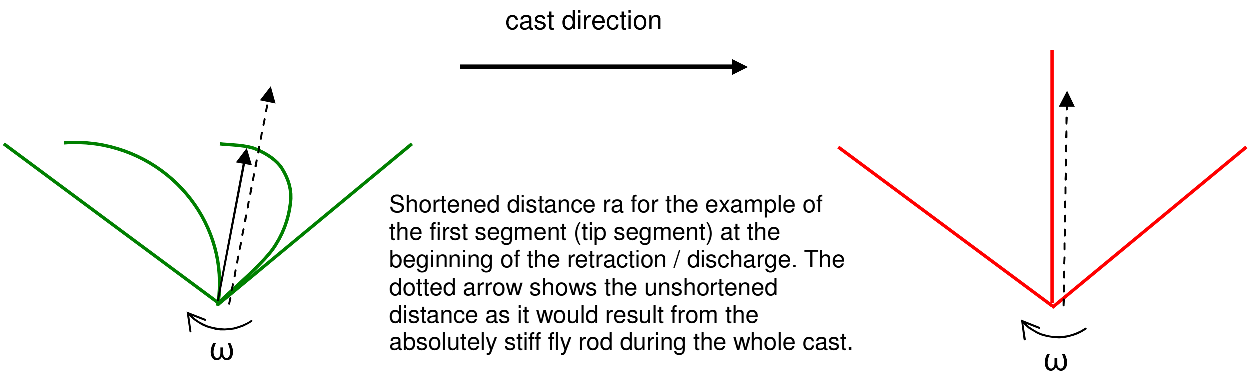
ηs = ½ * m * 13.294^2 / As,total = 88.36 / 78.658 = 1.12 ~ 1.0

<u>Annex 2:</u> Investigations of the influence of the mass of both fly rods

For both fly rods the same distribution of mass along the length of the rod is assumed. The geometry corresponds to that of a four segment SAGE 586 RPL+, as I used it during my cast sequence. The four parts (segments) of this fly rod have been measured and underlie my investigations. For the calculation of the moment of inertia each segment is assumed as a slim stick. The diameter for each segment is measured in the middle of the segments. The fly rod measured like this is shown by the following Figure:



The following Figures show the form of the mass of both fly rods during the cast.



Both fly rods differ mostly in the following properties:

a) Due to the shortening of the projection of the flexible fly rod the moment of inertia changes during the cast. Until the end of the retraction / discharge it reduces significantly, during the discharge it rises again. As the fly caster does not spend any work during the discharge (compare section E2 as well as 12. Conclusion) this rise does not lead to a rising effort. The moment of inertia of the absolutely stiff fly rod remains constant during the cast.

b) Due to the deflection the flexible fly rod does not distribute the angular kinetics which is introduced into the grip equally along the whole length. At the beginning of the retraction / discharge the upper part has an overproportional higher angular velocity than the lower part. As the caster does not change the angle of rotation in the grip he does not spend work / energy any more from this moment (compare section E2 as well as 12. Conclusion). Therefore this velocity of mass must not be attributed to the angular velocity. For that reason the flexible fly rod transfers most of the energy at the end of the cast. The absolutely stiff fly rod, in marked contrast, distributes the introduced energy equally along its length. Therefore it can not enhance the energy transfer efficiency towards the end of the cast.

Calculation of the momenta of inertia (investigation of a)

The deflection leads to a shortening of the vertical projection of the fly rod by about 1/3 of its length. (see 2nd conclusion). Between the 40° and 90° position it shortens by about 0.36 * 2.65m = 0.95 m. The vertical pitch of a segment measures to 95 cm / 2 = 47.5 cm. Between the 40° and 90° position the reduction of length is minor as the 47.5 cm must not be used for the whole cast. This is respected by the averaged reduction of length of 33cm referred to the vertical pitch of the tip segment (half length of segment ½ * 66 cm = 33 cm < 47.5 cm) and this estimation is not too high. For all further segments a linear shortening of the distance ra fits well the data measured by my cast (factor 3/3 for the 1st Segment, 2/3 for the 2nd segment, 1/3 for the 3rd segment). The grip segment is not shortened at all which is again ensuring that the investigation is not overestimating any effects. The mass m of the fly rods is contributed to each segment according to its cross sectional area A.

The values of this survey are collected in the following table:

Segment of the fly rod	**Tip segment = 1st segment**	**2nd segment**	**3rd segment**	**Grip segment = 4th segment**
Length per segment	**66 cm**	**66 cm**	**66 cm**	**66 cm**
Diameter per segment (measured in the middle)	**0.2 cm**	**0.45 cm**	**0.6 cm**	**0.9 cm**
Cross sectional area $A = \frac{1}{4} \pi d^2$	**0.031415 cm^2**	**0.1590 cm^2**	**0.2827 cm^2**	**0.6361 cm^2**
Moment of inertia Jseg for each segment i rotating around its own axis	**Jseg,i = 1/12 * m * l; m = mass of the segment, l = length of the segment**			
Distance ra between the center (=center of mass) of the segment and the rotational axis in the grip				
Absolutely stiff fly rod	**3*66cm+½*66cm = 231cm**	**2*66cm+½*66cm = 165cm**	**1*66cm+½*66cm = 99cm**	**½*66cm=33cm**
Flexible fly rod (40°-140° „average“ shortening)	**3*66cm+ 3/3*33cm = 198cm**	**2*66cm+ 2/3*33cm = 143cm**	**1*66cm+ 1/3*33cm = 88cm**	**½*66cm=33cm**
Moment of inertia J for each segment i rotating around the grip	**J,i = Jseg,i + m,i * ra^2,i; m = mass of the segment, ra = Distance of the vertical pitch to the rotational point of all segments, which is located in the grip (m*ra^2 corresponds to the part calculated according to the “law of Steiner”)**			
Absolutely stiff fly rod	**(1/12*m*66^2cm^2 +m*231^2cm^2) *0,031415 = 1687**	**(1/12*m*66^2cm^2 +m*165^2cm^2) *0,1590 = 4386**	**(1/12*m*66^2cm^2 +m*99^2cm^2) *0,2827 = 2873**	**(1/12*m*66^2cm^2 +m*33^2cm^2) *0,6361 = 923**
Flexible fly rod	**(1/12*m*66^2cm^2 +m*198^2cm^2) *0,031415 = 1243 m*cm^2**	**(1/12*m*66^2cm^2 +m*143^2cm^2) *0,1590 = 3309 m*cm^2**	**(1/12*m*66^2cm^2 +m*88^2cm^2) *0,2827 = 2292 m*cm^2**	**(1/12*m*66^2cm^2 +m*33^2cm^2) *0,6361 = 923 m*cm^2**

The moment of inertia for the whole system calculates to J = $\sum_i J, i$

Js(absolutely stiff fly rod) = **m*9.869 cm²**; Jf(flexible fly rod) = **m*7.767 cm²**

ΔJ = Jf / Js = m*7.767 cm² / m*9.869 cm² = 0.78

Difference between angular velocity and mass velocity (investigation of b)

This earlier described effect is varying with the distribution of mass of the fly rod and reduces as more as the mass is localized at the grip. With rising mass at the grip also the center of mass of the whole system shifts towards the grip and the influence of the mass of the upper part of the fly rod reduces, which is shifted behind. Without the mass of the roll an angular velocity ω which is reduced by 10 % would be a good estimation. This effect reduces remarkably with the mass of the roll and an angular velocity which is slowed down by about 2% would be a good estimation. <u>To be sure this remarkable smaller reduction of the angular velocity will form the basis for further investigations</u>. Including ω = φ / t the reduction of the angular velocity calculates to:

ωs(absolutely stiff fly rod) = 100/t; ωs² = 10.000/t²;

ωf(flexible fly rod)= 98/t; ωf² = 9.604/t²

Δω =ωf²/ ωs² = 0,96

Deceleration of both fly rods (energy of deceleration)

The mass of both fly rods has to be decelerated at the end of the cast. The energy which is necessary for that can be calculated by the physical principle of energy conservation. Respecting the mass of the fly rods the following relation holds for the energy during the whole cast, assuming that the fly line is not decelerated:

A(deceleration) + Arot + E(fly line) = 0

→ A(deceleration) = - Arot – E(fly line)

The kinetic energy of the fly line E(fly line) is neglectable. In fact during deceleration the fly line does not decelerate but elongates and produces a small force at the tip. All over E(fly line) is very small as compared to the kinetic energy of the fly rod and it can therefore be neglected. E(fly line) ~ 0. Then the following relation follows:

A(deceleration) ≈ - Arot.

At this point it should be remarked that if the contribution of the mass of the different fly rods is considered, also an additional effort to decelerate the fly rod must be taken into account. Even if this energy is negative the caster has to spend energy to stop the rod. For the absolutely stiff fly rod the whole energy has to be taken up by the caster in a very short time period. This is not the case for the flexible fly rod. Moreover the redistribution of mass in the flexible fly rod leads to a self deceleration process during the path of retraction and therefore to a largely reduced effort for the deceleration of the fly rod. This has to be taken into account too and leads to further advantages of the flexible fly rod.

Effort for the acceleration and the deceleration of the mass of both fly rods

The rotation energy Arot, which has to be spent by the caster for the acceleration of the mass of the fly rod calculates as follows:

Arot = ½ * J * ω².

For comparison we look at the quotient between the flexible fly rod and the absolutely stiff fly rod:

ΔArot = Arot (flexible fly rod) / Arot (absolutely stiff fly rod) = ΔJ * Δω

Acceleration and deceleration of their masses equals a transfer of the following amount of energy for the flexible fly rod compared to the absolutely stiff fly rod:

ΔArot (acceleration) = 0.78 * 0.96 = **0.75**

ΔArot (deceleration) = **0.78**

This means that in both directions, for the acceleration and for the deceleration the flexible fly rod exhibits advantages. For the sake of simplicity those both values could be multiplied to have an overall value that expresses the advantage of the flexible fly rod over the absolutely stiff fly rod just taking the mass of both rods into account. In summary that is

ΔArot = 0.75 * 0.78 = <u>0.58</u>

With other words: the effort for accelerating and decelerating the absolutely stiff fly rod as compared to the flexible fly rod is bigger by a factor of 1 / 0.58 = **1.71** just looking at the process performed on the own mass of both fly rods.

Consequences of the mass of both fly rods for my calculations

The mass relation between the fly line and the fly rod is about 1:8 (see foot note 30). If we estimate that the efficiency relation of 1.71 counts 8 times more than the gain of efficiency for casting the fly line (2.17 according to section E3) we get an overall advantage:

(1 * 2.17 + 8 * 1.71) / 9 = **1.76 = ηf / ηs (including the mass of both fly rods)**

This estimation of the mass influence shows that the basic results following from the comparison of both fly rods are not questioned by the mass of both fly rods. The gain in efficiency when the flexible fly rod is used might reduce slightly when taking its mass into account (2.17 → 1.76) , however the advantages of the flexible fly rod are not annihilated at all. The efficiency of the flexible fly rod is always higher than the efficiency of the absolutely stiff fly rod (if the flexible fly rod is deflected in a proper way) ! This estimation here also can be judged to be safe because a stiff (also virtually stiff) fly rod has more mass near its tip while the conical structure of the flexible fly rod leads to the mass distribution described above with only small contribution near the tip. This fact would rise the moment of inertia for the absolutely stiff fly rod. Therefore it can be assumed that the difference in efficiency which results to a factor of two for the efficiency as compared to the absolutely stiff fly rod is realistic !

In case of the absolutely stiff fly rod its mass can not contribute to an acceleration of the tip, because especially the moment of inertia can not reduce. This is different for the flexible fly rod ! By deflecting the mass of the flexible fly rod kinetic energy in transferred into the tip according to the effect of conservation of angular momentum (see section F1). The interplay between deflection and a mass that reduces in the direction to the tip, the flexible fly rod condenses or “pumps” the introduced kinetic energy into its tip. Therefore the velocity of the tip rises significantly as the experimental data of my cast sequence clearly show. In addition the flexible fly rod collects potential energy which additionally releases along the pathway of retraction additionally to the acceleration. The absolutely stiff fly rod does not have these intrinsic degrees of freedom and therefore just can accelerate the tip 1:1 (“one by one”) as introduced into the grip.

The relation v = r * ω clearly shows that the velocity is proportional to the angular velocity ω: v ~ ω. Therefore any change of the velocity of the tip also needs a change of the angular velocity in the grip. If the absolutely stiff fly rod gains its velocity v within a smaller rotation angle φ, then the rotational velocity ω needs to have (about) the same value as if an identical velocity v would have been reached along another rotational angle φ. As the rotational energy Arot mainly depends on the rotational velocity ω (which accounts to the energy by square, see above) the absolutely stiff fly rod can not reach a higher efficiency even for smaller rotation angles.

While the flexible fly rod can damp its mass with a counter deflection the absolutely stiff fly rod has to decelerate completely and suddenly over a short distance. According to the formula “force = work / distance (or angle)” it follows that the force that has to be spent for deceleration is as bigger as smaller the distance is. This can explain the hurting forearm as stated by casters of the absolutely stiff fly rod.

Annex 3: Explanations on the conservation of angular momentum by Dr. Franz-Josef Schmitt

When the distance of a rotating mass shifts towards the center of rotation then the velocity changes due to the conservation of angular momentum which is present in the system during that moment. The question if further angular momentum is introduced by external forces during the duration of the change of distance to the center of rotation is not relevant for the principle existence of this effect. At time point „x“ the angular momentum L is present in the system of the fly rod. If at a later time „x+n“ further angular momentum is introduced the part of the momentum which was already present in the systems must be conserved. If further angular momentum is introduced this will rise the velocity of the rod tip as an additional effect.

It is for example known how a person turning on an ice surface accelerates if during a pirouette the arms are attracted to the body. This leads to shortening of the average distance of the rotating mass from the center of rotation and as the momentary present angular momentum has to be conserved the velocity of the rotating mass must rise for compensation. This can also be observed.

The same is observable if a coin is rotating in a cone and approaches the center as more the coin approaches the center of the cone as faster it rotates.

The effect can be described mathematically by definition of the angular momentum $\vec{L}$ as the vector product of the distance vector from the center of rotation $\vec{r}$ und the vector for the momentum $\vec{p}$: $\vec{L} = \vec{r} \times \vec{p}$

The vector product is equal to the product of the absolute values of distance and momentum multiplied with the sinus of the angle α between the vectors $\vec{r}$ and $\vec{p}$:

$$\left|\vec{L}\right| = \left|\vec{r}\right|\left|\vec{p}\right| \sin\alpha$$

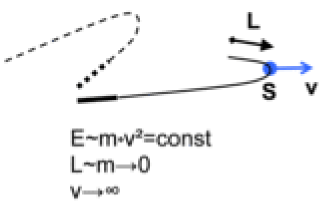
The angular momentum is therefore caused by the component of the dynamics which moves rectangular to the direction towards the center of rotation. If the distance to this center of rotation shortens necessarily the velocity and momentum of mass element rises to ensure that the momentary present angular momentum is conserved.

The effect is especially significant when the whip cracks. The phenomenon is transferable to energy conservation or conservation of angular momentum. From the conservation of angular momentum and energy the effect of the rising velocity can be exactly calculated:

For representation it is cited from Wikipedia:

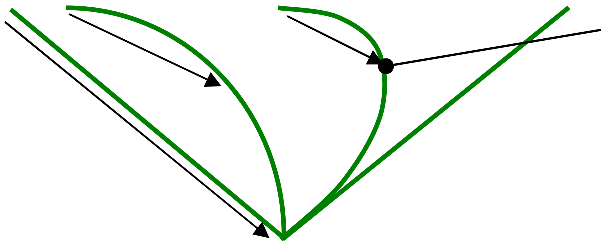
“The end of a whip can be accelerated to supersonics, if whipped correctly, resulting in the whip crack. The crack results from the formation of a loop which moves towards to tip of the whip with accelerated dynamics. When it opens up at the end of the line, supersonic velocity can be reached. The line tip reaches about double sound speed and is finally accelerated 50.000 times the apparent gravity. The theoretical description for this phenomenon was done by the physicist István Szabó, who cracked a whip in his lectures and afterwards noted the equations necessary for explanation onto the blackboard.

Physical relations



*At the lower point of the movement the whip is suddenly stopped. The whip line from the end of the grip until the maximum of curvation of the loop **S** is straight and quite at rest. Due to the centrifugal forces the whip line moves towards complete stretching. For that reason the loop **S** moves away from the grip axially and the rest of the whip line **L**, above the loop **S**, becomes smaller. The mass m of this rest* *line is proportional to its length L; therefore energy the mass of this rest line tends to zero. As the kinetic energy* $E = {}^{1}/_{2} \cdot m \cdot v^2$ *is constant the velocity v tends to infinity due to the conservation of energy. Practically the maximum velocity of the loop* ***S*** *, the tip of t e loop, respectively, is limited by internal and external friction*[59]*".*

The similarity of the deflection of the fly rod during Tobias´ cast leads to a similar contribution of this effect to the kinematics of the rod tip:



Differently to the whip the loop „S" does not move completely to the tip (see the Figure from Wikipedia above). Therefore the velocity v does not become „infinity" but rises by a certain factor. The positive properties for the transfer of energy into the fly line are used for this effect too what is not possible with an absolutely stiff fly rod.

In contrast to the whip the center of rotation does not completely move towards the tip. This results in a certain factor the velocity rises by but it does not tend "to infinity". The rise of the velocity is directly proportional to the shortening of the radius given by the distance from the tip that tends to stretch to the point of maximum curvation S which is a fictive „turning point". The effect becomes stronger by a reduction of the mass distribution – similar to the fly rod in direction to the tip. Then an overproportional smaller part of mass has to compensate for the reduced distance to the center of rotation and accelerates even more.

[59] http://de.wikipedia.org/wiki/Peitsche#Kinematik_des_Peitschenschlages